Resource Allocation in OFDM Based Wireless Relay Networks

by

Guftaar Ahmad Sardar Sidhu

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Approved, Thesis Committee:

Dr. Feifei Gao, Jacobs University Bremen

Dr. Jon Wallace, Jacobs University Bremen

Dr. Wei Liu, University of Sheffield

Date of Defence: July 18, 2012

School of Engineering and Science
I hereby confirm that this thesis is an independent work that has not been submitted elsewhere for conferral of a degree.

Bremen, 27th July 2012.

Guftaar Ahmad Sardar Sidhu
Dedications:

To my family
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Contents

Acknowledgment

Contents

Abstract

List of Tables

List of Figures

List of Acronyms

Chapter 1. Introduction

1.1 Overview of Relay Networks

1.2 Overview of OFDM

1.3 Basics of the Optimization Theory

1.4 Resource Allocation Problem

1.5 Contribution and Organization of the Thesis

Chapter 2. Resource Allocation in Multi-User Uplink Systems

2.1 Introduction

2.2 System Model

2.3 Resource Allocation Schemes

2.3.1 Problem Formulation

2.3.2 Joint Resource Allocation Scheme
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3.3 Proposed Low-Complexity Suboptimal Solution</td>
<td>30</td>
</tr>
<tr>
<td>2.4 Generalization to Multiple Relay Scenario</td>
<td>31</td>
</tr>
<tr>
<td>2.4.1 Joint Optimization</td>
<td>33</td>
</tr>
<tr>
<td>2.4.2 Suboptimal Algorithm</td>
<td>35</td>
</tr>
<tr>
<td>2.5 Simulation Results</td>
<td>35</td>
</tr>
<tr>
<td>2.6 Summary</td>
<td>40</td>
</tr>
</tbody>
</table>

### Chapter 3. Resource Optimization in Bidirectional Relay Networks 43

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1 Introduction</td>
<td>43</td>
</tr>
<tr>
<td>3.2 System Model</td>
<td>47</td>
</tr>
<tr>
<td>3.3 Problem Formulation</td>
<td>49</td>
</tr>
<tr>
<td>3.4 Joint Resource Allocation Scheme</td>
<td>50</td>
</tr>
<tr>
<td>3.4.1 Lagrange Dual Decomposition: Solving the Dual Function</td>
<td>51</td>
</tr>
<tr>
<td>3.4.2 Solving the Dual Problem with Sub-gradient Method</td>
<td>53</td>
</tr>
<tr>
<td>3.5 Step-Wise Low-Complexity Resource Allocation Scheme</td>
<td>54</td>
</tr>
<tr>
<td>3.5.1 Sub-carrier Allocation for Given Power Allocation</td>
<td>54</td>
</tr>
<tr>
<td>3.5.2 Tone Matching for Given Power Allocation and Sub-carrier Allocation</td>
<td>55</td>
</tr>
<tr>
<td>3.5.3 Power Allocation for Given Sub-carrier Allocation and Tone Matching</td>
<td>56</td>
</tr>
<tr>
<td>3.6 Simulation Results</td>
<td>56</td>
</tr>
<tr>
<td>3.7 Summary</td>
<td>59</td>
</tr>
</tbody>
</table>

### Chapter 4. OFDM Resource Allocation for Parallel Relay Communication 61

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Introduction</td>
<td>61</td>
</tr>
<tr>
<td>4.2 System Model</td>
<td>63</td>
</tr>
<tr>
<td>4.3 Cooperative Non-Orthogonal Transmission</td>
<td>64</td>
</tr>
<tr>
<td>4.3.1 Joint Optimization Algorithm</td>
<td>66</td>
</tr>
<tr>
<td>4.3.2 Suboptimal Algorithm</td>
<td>70</td>
</tr>
<tr>
<td>Chapter 5. Lifetime Maximization in Multi-hop Networks</td>
<td>82</td>
</tr>
<tr>
<td>-----------------------------------------------------</td>
<td>----</td>
</tr>
<tr>
<td>5.1 Introduction</td>
<td>82</td>
</tr>
<tr>
<td>5.2 System Model</td>
<td>85</td>
</tr>
<tr>
<td>5.3 Problem Statement</td>
<td>87</td>
</tr>
<tr>
<td>5.4 Lifetime Maximization Scheme</td>
<td>88</td>
</tr>
<tr>
<td>5.5 Simulation Results</td>
<td>92</td>
</tr>
<tr>
<td>5.6 Summary</td>
<td>94</td>
</tr>
</tbody>
</table>

Chapter 6. Resource Allocation in Cognitive Relay Networks

Chapter 7. Conclusions

Bibliography

List of Publications

Appendix A. Derivations of Closed-Form Expression for Power Allocation at the Relays

Appendix B. Derivations of the Optimal Power Allocations in Equations (5.16) and (5.17)
Abstract

The combination of relay transmission with orthogonal frequency division multiplexing (OFDM) technique is deemed as the candidate for the fourth generation (4G) wireless networks. This thesis addresses the resource allocation problem in OFDM based relay networks. Different scenarios of relay networks are considered, e.g., the multi-user relay networks, the multi-relay networks, and the cognitive radio (CR) relay networks. The resource management strategies are developed to analyse: how to optimally distribute available resources among different users, how resource optimization enhances the performance under cooperative communication, how to maximize the lifetime of multi-hop wireless sensor networks (WSNs), and how to improve CR transmission without degrading the performance of the primary network?

For each relay transmission scenario, two or more resources are optimized to enhance the system performance under various constraints. The resources include: the power allocation at the source terminal, the power allocation at the relay nodes, the sub-carrier allocation among different users, and the sub-carrier matching over different hops. Multi-user transmission adopts orthogonal frequency division multiple access (OFDMA) technique and is subject to separate power constraint at each terminal. Initially, the resource management issue in multi-user uplink relay transmission is discussed. Then, the similar resource allocation problem is solved when OFDMA multi-user system operates under the bidirectional relay transmission. The multi-relay dual hop transmission is optimized under different transmission schemes, specifically, the non-orthogonal transmission where all the
abstract

terminals transmit simultaneously in the second time slot and the time division multiple access (TDMA) based transmission where each relay transmits in the pre-defined time slot. The resource allocation in CR relay networks is considered which ensures that the interference caused by the secondary network is not harmful to the primary system. Convex optimization techniques are exploited to solve the problems for each relay transmission and the near optimal solutions are developed. Further, several suboptimal algorithms are also designed to reduce the computational complexity.
List of Tables

3.1 Complexity comparison of different algorithms . . . . . . . . . . . . . 57
List of Figures

1.1 Two hop relay transmission ........................................... 2
1.2 The OFDM symbol with cyclic prefix. ................................. 4
1.3 OFDM transceiver: block diagram ................................. 5
1.4 Organization of the thesis. ................................. 13

2.1 Multi-user single relay uplink system model with $M = 4$ and $K = 8$.
(Dashed lines show the pairing over two-hops) .......................... 19
2.2 Multi-user multi-relay uplink system model. ......................... 32
2.3 Rate versus SNR for $M = 10$ and $N = 1$. ......................... 37
2.4 Throughput versus the number of users for $N = 1$. .................. 38
2.5 Throughput versus SNR for $M = 10$ and $N = 5$. ................. 39
2.6 Throughput versus the number of users for $N = 5$. ................. 40
2.7 Throughput versus the number of relays for $M = 6$. ............... 41
2.8 Throughput versus the number of users and the number of relays for
JntSol. ............................................................................. 42

3.1 Bidirectional communication with OWRN. ......................... 44
3.2 Bidirectional communication with TWRN. ......................... 45
3.3 System model for an OFMDA aided multi-user TWRN. .......... 48
3.4 Throughput versus SNR for $M = 10$. ............................... 58
3.5 Throughput versus the number of users at SNR= 10 dB. ............. 59
3.6 Throughput versus SNR under anti-symmetric channels for $M = 2$
and $M = 10$, respectively. ............................................. 60
### List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>System model for dual hop parallel relay network</td>
<td>64</td>
</tr>
<tr>
<td>4.2</td>
<td>Throughput versus SNR for $N = 10$.</td>
<td>77</td>
</tr>
<tr>
<td>4.3</td>
<td>Convergence.</td>
<td>78</td>
</tr>
<tr>
<td>4.4</td>
<td>Rate versus SNR with orthogonal transmission.</td>
<td>80</td>
</tr>
<tr>
<td>4.5</td>
<td>Rate versus SNR with relay selection scheme.</td>
<td>81</td>
</tr>
<tr>
<td>5.1</td>
<td>A typical sensor network</td>
<td>83</td>
</tr>
<tr>
<td>5.2</td>
<td>Linear multi-hop network</td>
<td>84</td>
</tr>
<tr>
<td>5.3</td>
<td>System model of AF-DF alternating relay network</td>
<td>85</td>
</tr>
<tr>
<td>5.4</td>
<td>Lifetime convergence</td>
<td>92</td>
</tr>
<tr>
<td>5.5</td>
<td>Lifetime vs R-Req</td>
<td>93</td>
</tr>
<tr>
<td>5.6</td>
<td>Lifetime vs SNR</td>
<td>94</td>
</tr>
</tbody>
</table>
# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>UE</td>
<td>User Equipment</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
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<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
</tr>
<tr>
<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>3GPP</td>
<td>3rd Generation Partnership Project</td>
</tr>
<tr>
<td>4G</td>
<td>Fourth Generation</td>
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<tr>
<td>AF</td>
<td>Amplify and Forward</td>
</tr>
<tr>
<td>DF</td>
<td>Decode and Forward</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>ISI</td>
<td>Inter-Symbol Interference</td>
</tr>
<tr>
<td>ICI</td>
<td>Inter-Channel Interference</td>
</tr>
<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>IDFT</td>
<td>Inverse Discrete Fourier Transform</td>
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<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
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<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
</tr>
<tr>
<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
</tr>
<tr>
<td>TWRN</td>
<td>Two-Way Relay Network</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time Division Multiple Access</td>
</tr>
<tr>
<td>CR</td>
<td>Cognitive Radio</td>
</tr>
</tbody>
</table>
# List of Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>MU</td>
<td>Mobile User</td>
</tr>
<tr>
<td>RS</td>
<td>Relay Station</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>OWRN</td>
<td>One-Way Relay Network</td>
</tr>
<tr>
<td>MAP</td>
<td>Multiple-Access Phase</td>
</tr>
<tr>
<td>BCP</td>
<td>Broadcast Phase</td>
</tr>
<tr>
<td>SN</td>
<td>Source Node</td>
</tr>
<tr>
<td>DN</td>
<td>Destination Node</td>
</tr>
<tr>
<td>MRC</td>
<td>Maximum Ratio Combining</td>
</tr>
<tr>
<td>LAN</td>
<td>Local Area Network</td>
</tr>
<tr>
<td>WSN</td>
<td>Wireless Sensor Network</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>FCC</td>
<td>Federal Communications Commission</td>
</tr>
<tr>
<td>SU</td>
<td>Secondary User</td>
</tr>
<tr>
<td>PU</td>
<td>Primary User</td>
</tr>
<tr>
<td>SSN</td>
<td>Secondary Source Node</td>
</tr>
<tr>
<td>SRN</td>
<td>Secondary Relay Node</td>
</tr>
<tr>
<td>PDN</td>
<td>Primary Destination Node</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

In this chapter, we present overview for relay networks, OFDM system, and the resource allocation problem for OFDM based relay networks. We also briefly provide the basics of convex optimization theory. At the end, we describe the major contributions of this work.

1.1 Overview of Relay Networks

A typical cellular wireless system consists of a Base Station (BS) and a number of User Equipments (UEs). Due to the rapid development of multimedia application, the current wireless communication systems demand high speed data transmission and better coverage. To fulfil the increasing throughput requirements at higher Quality of Service (QoS), the future wireless systems will be more demanding. The multiple-antenna technologies i.e., Multiple-Input Multiple-Output (MIMO) have been recognized as important solutions [1]. A significant diversity and multiplexing gain can be obtained by using multiple antennas at transmitter and/or receiver [2]. The net effects of these gains are the improvements in terms of network capacity and wireless link quality. However, implementation of multiple antennas in mobile devices may not be possible due to their small sizes and power limitations.

A high data rate could be achieved if large bandwidth is allocated to the
1.1 Overview of Relay Networks

wireless system. A direct result of increased bandwidth is that the required center frequency is high, which significantly increases the signal attenuation during the transmission. Further, due to the path loss of radio propagation, the received signal power decreases with the increasing distance between transmitting and receiving nodes. One of the solution to this coverage and capacity problem is to decrease the cell sizes by installing more BSs in the network, thus making the distance between the BS and the cell edge users smaller. However, the higher cost of deploying a BS makes this option less appealing for the network operators. As a second solution, a better range extension and throughput enhancement could be achieved with increasing the transmit power. The benefit of this strategy is limited by the increased interference to the co-existing networks and is also not recommended due to health issues.

In the recent years, relay networks have been proposed to provide a promising solution to capacity and coverage problem in wireless networks. A relay is a node in communication systems that receives a signal from a transmitting node and then forwards this signal to the next node. Due to limitations of current radio technology, a node cannot transmit and receive simultaneously in the same band through the same antenna element. As a result, the relay node must operate in half-duplex mode. Therefore the relay-assisted transmission is carried out in two phases: in the first phase the relay receives the signal from the transmitter and in the second phase it transmits to the destination. Figure 1.1 depicts an example of the relay-assisted transmission in a three-terminal network. The destination can

![Figure 1.1: Two hop relay transmission](image)

combine the transmissions received by the source and the relay or can only receive
the signal transmitted by the relay, if, e.g., the channel gain of the direct link is degraded due to obstacles or/and path losses.

The history of relay channels can be dated back to the 70s [3]-[5], when Cover [5] proposed several relaying schemes and studied the corresponding system capacity. Despite its early discovery, research on relay network did not become popular until the topic re-attracted the research community in late 90s after the work presented in [6] and [7]. Later Laneman applied the theory to cooperative communication scenario to develop low-complexity cooperative diversity protocols in [8].

The multihop communication has been proposed in the emerging Fourth Generation (4G) standards such as Long Term Evolution Advanced (LTE-Advanced) developed under 3rd Generation Partnership Project (3GPP) [9] and IEEE 802.16j to fulfil the high data rate requirements for a comparatively low cost. Depending on the distance between the two communication terminals and the nature of surrounding particles, one or more relays can be used to assist the communication for coverage enhancement. Further, cooperative diversity has been proposed as an alternative solution to the multiple antennas where the transmission between the source and the destination is performed with the assistance of a number of relay nodes each with one antenna [10]. In particular, the source node selects one or more relays and forwards its data to them. Then, the source and relays coordinate their transmissions in such a way that the maximum multiplexing/diversity gains are achieved at the destination node. Generally speaking, there are two modes of relaying transmission:

- **Amplify-and-Forward (AF):** In AF mode the received signal at the relay node is directly amplified and is retransmitted to the next station. The AF scheme is easy to implement and can be easily upgraded to the new coding schemes. However, the noise effect is propagated to the next node which reduces the effective signal to noise ratio (SNR) in the final destination.

- **Decode-and-Forward (DF):** In DF mode the relay decodes the information bits and re-encodes the data for further transmission. The main advantage of DF protocol is that the noise effect can be removed if the information bits are
correctly decoded. However, if wrong detection appears, the error will be propagated into the next station. Moreover, DF requires energy consumption and time delay for decoding and encoding the massage.

Based on the topology, a network can have a single or multiple relay nodes to transmit data from the source to the destination. In single relay system, relay receives a signal from the source and forwards it to the destination after processing. The multi-relay networks could be further categorized into two types: parallel relay networks and multi-hop networks. In parallel relay networks, more than one nodes are deployed between two communication terminals such that the source transmits to all relays at the first hop and the relays forward this information to the sink at the second hop. The relays do not communicate with each other under the parallel relay model. The dual hop transmission (single or multi-relay) may not be sufficient to provide ubiquitous data rate between two nodes if the end-to-end SNR is very low. A promising solution is to place more than one interconnected relay nodes such that the transmission from the source to the destination occurs in multiple hops.

1.2 Overview of OFDM

OFDM is a commonly used multi-carrier transmission technique to combat the frequency selective channels as well as the inter-symbol interference (ISI). The history of OFDM could be traced back to the mid 60’s, when Chang proposed
1.2 Overview of OFDM

his idea of the parallel transmissions over multiple channels [11]. He developed a principle for transmitting messages simultaneously through orthogonal channels that is free of inter-channel interference (ICI) and the ISI. In OFDM systems, the available bandwidth is divided into $K$ sub-channels (sub-carriers) and data is transmitted in parallel on these sub-carriers. Different modulation schemes can be used for different sub-carriers. To combat ISI caused by channel time spread, a cyclic prefix (CP) that duplicated last portion of an OFDM block is inserted in the front of OFDM block [12]. As long as the length of CP is greater than the impulse response of the channel, perfect orthogonality among sub-channels is achieved and the effect of ISI caused by the arrival of different OFDM symbols with different delays is completely eliminated. A block structure of CP based OFDM symbol is shown in Fig. 1.2.

To transmit data via OFDM symbol, a serial to parallel conversion of bit stream is performed in the OFDM transmitter. The transmitter transforms this parallel bit stream into the time domain using an Inverse Discrete Fourier Transform (IDFT). In practical systems, the Inverse Fast Fourier Transform (IFFT) algorithm is used due

![Figure 1.3: OFDM transceiver: block diagram](image-url)
1.3 Basics of the Optimization Theory

The design of wireless systems often involve the optimization of an objective subject to certain resource constraints. An optimization problem with arbitrary equality and inequality constraint can be written as [13]

$$\min_x f_o(x)$$

subject to $$f_i(x) \leq 0, \ 1 \leq i \leq m,$$

$$h_i(x) = 0, \ 1 \leq i \leq p,$$

where $$x$$ is the optimization variable, $$f_o$$ is the objective function, $$f_1, \ldots, f_m$$ are the $$m$$ inequality constraint functions, and $$h_1, \ldots, h_p$$ are the $$p$$ equality constraint functions.

The optimization problems could be grouped into different classes depending on the nature of the objective function and the constraint functions. The problem is called linear programming if the objective and all constraint functions are linear. The linear programming problems can be solved effectively using the well known Dantzig’s simplex method, the criss-cross algorithm, and the recently developed interior point methods [13]. The problem is called non-linear optimization if the objective and/or constraint functions are non-linear and non-convex [14].

The convex optimization theory gives an easy solution if the problem is, or can be transformed into, convex problem. A problem is called a convex optimization problem if the objective function $$f_o$$ and the inequality constraint functions $$f_1, \ldots, f_m$$ are convex and the equality constraint functions $$h_1, \ldots, h_m$$ are affine. Convex optimization problems can be optimally solved either in closed form
or numerically, since they have a global minima. However, most of the engineering problems are not necessarily convex in nature, therefore some reformulations are done to state the problem in standard convex form. A common technique to formulate a problem into convex one, is by relaxing the problem from removing some constraints such that it becomes convex.

If the convex optimization problem does not have any constraint function the efficient iterative algorithms have been developed to find the global optimal solution e.g., the gradient descent method, the steepest descent method, and the Newton’s method. The Lagrange duality theory is one of the fundamental tool for solving the constrained convex optimization problems. The basic idea behind the Lagrange duality is to take the constraints of problem (1.1) into account by augmenting the objective function with a weighted sum of constraint functions. The Lagrangian is defined as

\[
L(x, \lambda, \nu) = f_o(x) + \sum_{i=1}^{m} \lambda_i f_i(x) + \sum_{i=1}^{p} \nu_i h_i(x),
\]

where \( \lambda = \{\lambda_i, \forall i\} \) and \( \nu = \{\nu_i, \forall i\} \) are called the Lagrange multipliers or dual variables. The Lagrange dual function is defined as the minimum value of the Lagrangian over \( x \)

\[
g(\lambda, \nu) = \inf_{x \in D} L(x, \lambda, \nu).
\]

The dual function is always convex in terms of dual variables regardless of the convexity of the original function. The dual optimization problem is given by

\[
\max_{\lambda, \nu} \quad g(\lambda, \nu)
\]

subject to \( \lambda_i \geq 0, \nu_i \geq 0, \forall i \).

The duality gap is defined as

\[
d = f_o(x^*) - g(\lambda^*, \nu^*) \geq 0,
\]

where \( f_o(x^*) \) and \( g(\lambda^*, \nu^*) \) are the optimal primal and dual values, respectively. It has been shown in [13] that if the original problem is convex, under some constraint conditions, the duality gap reduces to zero (i.e., strong duality) and (1.5) holds with
1.3 Basics of the Optimization Theory

equality. In other words, both the primal and the dual optimization problems have the same solution. Therefore, one way to solve the original problem is to solve, instead, its associated dual problem.

For the optimal primal and dual points \((x^*, \lambda^*, \nu^*)\) with zero duality gap, the Karush-Kuhn-Tucker (KKT) conditions are defined as [13]

\[
\begin{align*}
  f_i(x^*) &\leq 0, \quad i = 1, \ldots, m \\
  h_i(x^*) &= 0, \quad i = 1, \ldots, p \\
  \lambda_i^* &\geq 0, \quad i = 1, \ldots, m \\
  \lambda_i^* f_i(x^*) &= 0, \quad i = 1, \ldots, m \\
  \nabla f_o(x^*) + \sum_{i=1}^{m} \lambda_i \nabla f_i(x^*) + \sum_{i=1}^{p} \nu_i \nabla h_i(x^*) &= 0
\end{align*}
\]

For differentiable \(f_o\) and \(f_i, \forall i\), the KKT conditions are necessary for the optimality, and for the convex primal problem, the KKT conditions are also sufficient for the points to be primal and dual optimal [13]. In summary, for any convex optimization problem with differentiable objective and constraint functions, any points that satisfy the KKT conditions are primal and dual optimal. The dual decomposition technique has been widely used to solve the optimization problems and is appropriate when the optimization problem has a coupling constraint such that, when relaxed, the problem decouples into several sub-problems. The idea behind the technique is to decompose the original complicated problem into solvable simple sub-problems.

The KKT conditions are the necessary and sufficient conditions for the convex optimization problems. However it may not be always possible to find the closed form solution from the KKT conditions. Thus, other promising algorithms have been developed for the constrained convex optimization problems for example the interior point methods and the sub-gradient methods [13]. Interior point method is a search algorithm which could be adopted to transform the constrained optimization problem into simplified unconstrained problems. For non differentiable objective functions, the sub-gradient method is used. The sub-gradient method is generally used with decomposition techniques to solve the large optimization problems.
1.4 Resource Allocation Problem

The motivation behind this research project is to develop efficient resource utilization algorithms for OFDM based relay network. Although optimal resource allocation techniques have been developed for traditional point to point OFDM networks, these techniques cannot be directly applied to the relay networks due to the strong coupling between different hops. Optimization tries to find the best strategy to use the resources (optimization variables) such that a performance metric (objective functions) is improved under certain system limitations (constraints function).

Resource allocation in OFDM systems, for example the power allocation and the sub-carrier assignment, are formulated into an optimization problem with an objective to optimize system performance such as the system throughput subject to various constraints, e.g., the power limitation of the batteries of transmitting nodes. The optimization techniques, such as the dual decomposition, the primal decomposition, and the sub-gradient methods are used to optimally solve these resource allocation problems. In this section, we give a general overview of resource allocation in OFDM based relay networks. Detailed review of the existing works will be presented in the next chapters.

The optimal power allocation is known to enhance the system performance in traditional point to point OFDM systems and is achieved by applying the water-filling algorithm such that large amount of power is loaded on the sub-carriers which have high SNR values. The power allocation problems are formulated in two different ways. First, the objective is to maximize the system throughput given the constrained transmission power. The second aims to minimize the transmission power while providing a minimum QoS.

In relay aided communication, the fading of different channels are mutually independent such that the sub-carriers which experience deep fading over one hop may not face deep fading over the next hop. With this fact, the reception and transmission over the same sub-carrier is clearly sub-optimal and could result in severe performance loss. Therefore, pairing the sub-carriers from different hops
is important to enhance the transmission rate. The first work on the sub-carrier pairing problem was reported in [15] where the OFDM modulated single user dual-hop relay network is discussed. It was proved that the system throughput can be increased if the sub-carriers on the two hops are coupled in the order of their channel magnitudes. This work was further exploited in [16], where a suboptimal carrier coupling algorithm was presented for multi-relay scenario. However, both [15] and [16] did not consider the power allocation which may change the effective channel magnitude. In [17], the authors proposed a joint power allocation and sub-carrier pairing algorithm for single user single relay network under different power constraints.

On the other hand in multiple user scenario, the existence of the users at different locations introduces multi-user diversity where a particular sub-carrier undergoes different fading attenuation for different users. To exploit the multiuser diversity and to avoid the inter-user interference, Orthogonal Frequency Division Multiplexing (OFDMA) has been widely adopted in the wireless communication networks. In OFDMA transmission, each user is assigned a disjoint set of sub-carriers such that each sub-carrier is assigned to a unique user based on a given criteria, e.g., to a user which results in least attenuation. The sub-carrier allocation has almost always been considered with the other resource allocation in OFDMA networks. Generally speaking, the resource allocation problem can be divided into two categories: the overall system throughput maximization problem and the fair resource allocation problem. In throughput maximization problem, the sub-carriers are allocated in such a way that the sum rate of all users is maximized. However, this scheme lacks the fairness among user because more sub-carriers could be allocated to the users which have good channels. Under the fair resource allocation problem, each users is granted a fair share of resources. However, on the other hand, this schemes can have severe negative effects on the overall performance because more resource will be gone to the users which are in bad channel conditions. In this thesis, we focus on the overall system performance enhancement problem to avoid the wastage
of resources by allocating sub-carriers to the nodes with large attenuations.

In multi-relay OFDM networks, the optimal relay assignment should also be included in the resource allocation problem. Similar to multi-user diversity, sub-carriers have independent fading for different relaying nodes. In conventional OFDM systems, the system performance is optimized through optimal power allocation among different sub-carriers. However, the resource allocation in OFDM based relaying transmission is more challenging. A deep faded sub-carrier over the first hop for one relay station may be a good candidate over the second hop for the same relay. Thus, a careful power distribution, sub-carrier pairing, and relay assignment policy is required for the guaranteed performance of the system. All these resources are strictly coupled with each other, for example, changing the power distribution will certainly change the sub-carrier pairing over the two hops and this can result in different relay assignments. Thus, designing the joint resource allocation schemes is necessary for the promising advantages of relay networks and is the main aim of this thesis’s work.

1.5 Contribution and Organization of the Thesis

The relay networks have gained much interest due to their capability of enhancing the communication reliability and enlarging the transmission range [18], [19]. Meanwhile, multi-carrier transmissions are known to combat the frequency selective fading channels and, when combined with the relay transmission, can provide improved performance through adaptive resource allocation. Hence, various researches on multi-carrier aided relay networks have been carried out during the past few years, for example, channel estimation [20], precoder design [21], and throughput analysis via resource allocation [17].

In this thesis, we study resource allocation problems for relay enhanced networks. The main objective of the research is to develop optimal and computational efficient resource allocation algorithms for various relay transmission schemes. In the context of relay transmission, we consider five different schemes: 1)
uplink relay transmission, 2) bidirectional relay transmission, 3) multi-relay dual-hop transmission, 4) multi-hop transmission, and 5) cognitive relay transmission. The detailed description of different schemes will be presented in the next chapters. For each transmission scheme, the aim is to enhance the system performance, for example the system throughput or network lifetime, subject to power and the other transmission specific constraints. Depending on the transmission strategy, different algorithms are developed to optimize two or more resources introduced in section 1.4.

In chapter 2, a joint resource allocation problem is formulated for multi-user multi-carrier uplink relay transmission system. The objective function is to maximize the sum-rate over joint sub-carrier allocation, sub-carrier pairing, and power allocation under the individual power constraints at each transmitting node. Asymptotically optimal and computationally efficient algorithms are developed. Further, the algorithm is extended to multiple relay scenario where the optimal sub-carrier to relay assignment is also obtained.

In chapter 3, we develop resource allocation algorithms for OFDMA assisted two-way relay network. The two way relaying promises to overcome the rate loss problem caused due to the half duplex relay transmission, however, makes the resource allocation problem more challenging. We develop a scheme which jointly optimizes the sub-carrier assignment to the pre-defined user pairs, the tone matching over the two phases of the transmission, and the power allocation over the sub-carriers subject to the limited availability of the power budgets at the user/relay nodes and the OFDMA constraints. A low complexity algorithm is also designed which shows its comparable performance via simulation results.

In chapter 4, we explore a dual hop multi-relay network where all the relays receive/transmit information on a shared channel. First, a concurrent transmission is considered, where all the relays simultaneously transmit data in the second hop. The resource allocation algorithms are designed such that the power allocation at the source node, the beamforming at the relay nodes, and the sub-carrier pairing over the
two hops are jointly optimized via convex optimization techniques to maximize the end-to-end system throughput. Secondly, a Time Division Multiple Access (TDMA) based orthogonal transmission is considered. The power loading at the source/relay nodes and the sub-carrier matching over the two hops in each time slot are optimized in the developed algorithm. Finally, simulation results are provided to corroborate the proposed studies.

The chapter 5 considers the multi-hop transmission scheme where more than one intermediate relaying nodes are deployed for reliable transmission in wireless sensor networks. The relay nodes in the sensor networks are consist of small batteries such that the replacement or recharging of batteries is not possible. Thus, we develop lifetime maximization algorithms where the optimization problem is formulated considering both the initial energy and the power constraint at each node. To reduce the energy utilization for decoding and encoding in DF relays and to minimize the noise enhancement with AF relay transmission, we consider a mixed AF-DF relay transmission. Convergence and the performance of the proposed iterative algorithm is shown by simulations.

In chapter 6, we investigate the resource allocation problem in cognitive
1.5 Contribution and Organization of the Thesis

relay transmission. The cognitive radio (CR) has been proposed to improve spectrum utilization by exploiting the less used spectrum in a dynamically changing environment. In the proposed algorithms, the secondary user’s throughput is maximized through the power allocation (at the secondary source node and the secondary relay node) and the sub-carrier pairing (on two hops of secondary system) subject to various power constraints. Meanwhile, the interference from both the secondary source and the secondary relay node to the primary receiver is kept within an acceptable limit. The numerical examples demonstrate enhanced performance compared to trivial cognitive relay resource allocation algorithms.

In chapter 7, we conclude our work by highlighting the major contributions of the thesis. An overview of the thesis organization is shown in Fig. 1.4.
Chapter 2

Resource Allocation in Multi-User Uplink Systems

In this chapter, we propose a joint resource allocation scheme for relay aided uplink multi-user transmission with a single destination node. To reduce the computational complexity, we present a suboptimal algorithm which sacrifices very little on the performance. Moreover, we extend the proposed algorithms to the very general relay network with multi-user and multi-relays. The proposed algorithms could provide improved performance over the existing algorithms.

2.1 Introduction

OFDMA is a spectrally efficient multiplexing technique used in various wireless systems such as IEEE 802.11, IEEE 802.16 and is a promising candidate for future cellular networks e.g., accepted for IEEE 802.16j and LTE-Advanced. Disjoint sets of sub-carriers could be formed from OFDM symbol. OFDMA employs OFDM and has an innate feature of exploiting the frequency selectivity enabled multi-user diversity such that the multiple users transmit their information simultaneously using the different sub-carriers without inter-user interference. Based on the quality of wireless channels, the power allocation over sub-carriers and the set of sub-carriers
allocated to a node can be optimized.

Similar to traditional point-to-point multi-user OFDMA system[22], an OFDMA relay network is defined as a broadband relay network where a sub-carrier can be assigned to only one transmitting node at any time. The promise of simple receivers and high system performance has landed OFDMA relay networks as one of the prime multiple-access schemes for future generation broadband wireless networks, e.g., 802.16j. In traditional OFDMA systems, it is possible to optimize the system performance from carefully assigning sub-carriers and power among different users [23, 24]. However, the resource allocation in OFDMA relay networks is more challenging. A sub-carrier with low SNR over the first hop for one node may be a good candidate over the second hop for the same or another node, thus, a careful power distribution and sub-carrier allocation policy is mandatory for promising performance of OFDMA relay networks.

The power allocation between source and relay node play an important role in performance enhancement [25]. In OFDM relay networks, the independent nature of the channels over different hops motivates to consider the sub-carrier pairing at relay nodes[15]. In [17], the authors proposed a power allocation and sub-carrier pairing algorithm for single user single relay network under separate power constraints at source and relay nodes. The algorithm follows a step wise approach. In the first step, sub-carriers are paired according to the channel gains assuming an equal power allocation policy. In the second step, an alternate power allocation policy is adopted such that the source (relay) power is optimized for the known relay (source) power. Further, a joint source and relay power allocation scheme is also presented under the total power constraint.

On the aspects of the multi-user scenario, resource optimization in relay networks has been studied in [27]–[29] for downlink scenario. For example, the resource allocation in OFDM based multi-user multi-hop relay network was considered in [27]. The joint sub-carrier allocation and power loading in OFDMA based relay networks, with or without fairness, have been studied in [28]. The
2.1 Introduction

authors in [29] investigated rate, power, and sub-carrier allocation problem in multi-user scenario to maximize the system throughput under a total power constraint.

For downlink transmission, the BS could distribute the power and allocate the sub-channels to different users in a centralized manner. Unfortunately, the results from [27]–[29] cannot be directly applied to the uplink scenario due to the distributed nature of the users, that is, each user has an individual power constraint and could not share power among themselves. In this case, the resource allocation is more complicated and challenging, and only a few works have been reported yet [30]–[33]. The sub-carrier allocation to source/relay nodes in multi-user multi-relay OFDMA system is considered in [30]. The authors in [31] studied the sub-carrier and power allocation problem in a DF based multi-user relay network. The work in [32] considered the joint sub-carrier allocation and power allocation problem in AF enhanced OFDMA system and an iterative solution through mathematical decomposition techniques is obtained. Moreover, a suboptimal algorithm, where the sub-carriers over two hops are paired according to their magnitude, is also proposed. More recently, a joint resource allocation problem for uplink multi-user relay transmission is formulated in [33], and a suboptimal resource allocation algorithm is developed by fixing one source when optimizing the others.

In this chapter, we first consider the resource allocation problem in a single relay aided dual hop multi-user uplink system. Following questions are answered: How to allocate disjoint sets of sub-carriers among different users? How to find the optimal sub-carrier matching over two hops? What is the optimal power distribution at each transmitting node? And the most important is how to coordinate among all these for a joint optimization? To provide a complete study for a general multi-node uplink relay network, we extend whole principle to multi-relay case as well. With the existence of multiple relay nodes in an OFDMA system, different sets of sub-carriers could be allocated over two hops which gives us both more design freedoms and typically higher design complexity.
2.1 Introduction

The main contributions of the chapter are summarized as follows:

- We formulate a joint power allocation, sub-carrier allocation, and sub-carrier pairing problem which targets to maximize the overall system throughput subject to individual power constraints. Unlike the previous reported works [30]–[31] which optimize resources separately, we provide a unified formulation which includes sub-carrier pairing, sub-carrier allocation, and power allocation under the same optimization.

- A joint solution is obtained from the dual decomposition technique and a near optimal solution is found. The optimality of the proposed solutions is validated when the number of the OFDM sub-carriers is large [35], which is normally satisfied by the practical transmissions.

- We also propose a suboptimal algorithm to trade the performance for the computational complexity by separately finding parameters one at a time. The simulation results show less degraded performance of the suboptimal algorithm.

- For multi-relay multi-user network, the joint relay selection, sub-carrier allocation, sub-carrier matching, and power allocation is obtained in the second part of this chapter. Unlike [32], where a particular user can only be served by at most one relay node, we consider a more general scenario where different mobile users transmit the signals over different sub-carriers to more than one relay nodes, whereas a particular relay can serve more than one users.

This chapter is organized as follows. In Section 2.2, we present the system model of relay aided multi-user transmission. In Section 2.3, we formulate a unified resource allocation problem and propose a joint optimization algorithm from the dual composition method. A suboptimal algorithm is also designed to trade the performance for complexity. Extension to multi-user multi-relay scenario is then provided in Section 2.4. Numerical results are presented in Section 2.5 to corroborate the proposed studies, and conclusions are made in Section 2.6.
2.2 System Model

Consider an uplink multi-user transmission, where \( M \) mobile users (MUs) communicate with a BS through a single relay station (RS), as shown in Fig. 2.1. All MUs are sufficiently far from BS such that there is no direct communication link [17]. We assume that each node is equipped with only one antenna that cannot transmit and receive simultaneously. Meanwhile, we adopt multi-carrier transmission with \( K \) sub-carriers for all the channel links, i.e., between MU and RS it is OFDMA while between RS and BS it is OFDM.

If the \( k \)-th sub-carrier is assigned to the \( m \)-th MU in the first hop, then the received signal at RS is

\[
y_{m,k}^{\text{RS}} = p_{m,k} h_{m,k} x_{m,k} + w_{m,k}, \quad m \in \{1, \ldots, M\}, \quad k \in \{1, \ldots, K\},
\]

(2.1)
2.2 System Model

where $h_{m,k}$ denotes corresponding channel coefficient, $x_{m,k}$ and $p_{m,k}$ are the information symbol and power assigned to the $k$-th carrier from the $m$-th MU, and $w_{m,k}$ is the additive white Gaussian noise (AWGN) with variance $\sigma_{m,k}^2$.

Suppose that the signal received at RS over the $k$-th sub-carrier will be forwarded to BS over the $j$-th sub-carrier, after being scaled by a factor

$$\rho_j = \sqrt{\frac{q_j}{p_{m,k} |h_{m,k}|^2 + \sigma_{m,k}^2}},$$

where $q_j$ is the power allocated by RS on the $j$-th sub-carrier. Denote the channel coefficient between RS and BS on the $j$-th sub-carrier as $g_j$. The signal received at BS is then

$$y_{BS}^j = g_j p_{m,k} h_{m,k} x_{m,k} + g_j w_{m,k} + w_j, \quad j \in \{1, \ldots, K\},$$

where $w_j$ represents the additive noise at BS with the variance $\sigma_j^2$.

The received SNR on the sub-carrier pair $(k;j)$, if assigned to the $m$-th MU, is expressed as

$$\text{SNR}_{m,(k;j)} = \frac{|h_{m,k}|^2 |j|^2 |g_j|^2 p_{m,k}}{|\rho_j^2 |g_j|^2 \sigma_{m,k}^2 + \sigma_j^2},$$

and the corresponding data rate is

$$r_{m,(k;j)} = \frac{1}{2} \log_2 \left( 1 + \frac{|h_{m,k}|^2 |j|^2 |g_j|^2 p_{m,k}}{|\rho_j^2 |g_j|^2 \sigma_{m,k}^2 + \sigma_j^2} \right),$$

with $a_{m,k} = \frac{|h_{m,k}|^2}{\sigma_{m,k}^2}$ and $b_j = \frac{|g_j|^2}{\sigma_j^2}$. The $\frac{1}{2}$ factor counts for the two hops that are used for one complete transmission.

To assist the mathematical discussion, we define $\pi_{(k,j)} \in \{0, 1\}$ as the binary variable for the sub-carrier pairing:

$$\pi_{k,j} = \begin{cases} 
1, & \text{if the } k\text{-th sub-carrier of the first hop is paired with} \\
0, & \text{otherwise.} 
\end{cases}$$
2.3 Resource Allocation Schemes

In order to relate the sub-carrier pair \((k, j)\) to a specific user, we further define the binary variable \(\tau_{m,(k,j)} \in \{0, 1\}\), such that

\[
\tau_{m,(k,j)} = \begin{cases} 
1, & \text{if sub-carrier pair } (k, j) \text{ is allocated to the } m\text{-th user,} \\
0, & \text{otherwise.}
\end{cases}
\] (2.6)

With the above definition, the overall system throughput can be expressed as

\[
C = \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau_{m,(k,j)} r_{m,(k,j)},
\] (2.7)

and will be used as our main objective function throughout the chapter.

2.3 Resource Allocation Schemes

Our target is to jointly optimize the sub-carrier allocation, sub-carrier pairing, and the power allocation such that the end-to-end system throughput is maximized under individual power constraints of MUs and RS.

2.3.1 Problem Formulation

Mathematically, we need to optimize over the variables \(\pi = \{\pi_{k,j}\}, \tau = \{\tau_{m,(k,j)}\}, p = \{p_{m,k}\} \text{ and } q = \{q_j\}\) for all \(m = \{1, ..., M\}, k = \{1, ..., K\}, j = \{1, ..., K\}\). Let \(P_m\) and \(Q\) be the total powers of the \(m\)-th MU and RS, respectively. The
optimization can be formulated as

\[
\max_{\tau, p, q} \quad C \\
\text{s.t.} \quad \sum_{k=1}^{K} \pi_{k,j} = 1, \forall j, \quad \sum_{j=1}^{K} \pi_{k,j} = 1, \forall k, \\
\sum_{m=1}^{M} \tau_{m,(k,j)} = 1, \forall (k,j), \\
\sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau_{m,(k,j)} p_{m,k} \leq P_m, \forall m, \\
\sum_{j=1}^{K} q_j \leq Q, \\
p_{m,k} \geq 0, \quad q_j \geq 0, \quad \forall m, k, j.
\] (2.8)

The first and the second constraints come from the fact that each sub-carrier on the first hop can be coupled with one and only one sub-carrier in the second hop and vice versa. The third constraint guarantees the exclusive allocation of the sub-carrier pair \((k,j)\) to one user only. However more than one sub-carrier pairs can be allocated to a particular user. Other constraints represent individual power constraint at each node.

Unfortunately, problem (2.8) involves a mixed binary integer programming [34] which is in general difficult to solve. Thanks to [35], the duality gap of (2.8) is proved to be asymptotically zero if the number of sub-carriers \((K)\) is sufficiently large, and a rigorous investigation can be found in [36]. Thus we can solve the dual problem instead of the original problem.

The dual function of (2.8) can be defined as [13]

\[
D(\nu, \lambda) = \max_{\pi, \tau, p, q} \quad L(p, q, \pi, \tau, \nu, \lambda) \\
\text{s.t.} \quad \sum_{k=1}^{K} \pi_{k,j} = 1, \forall j, \quad \sum_{j=1}^{K} \pi_{k,j} = 1, \forall k, \quad \sum_{m=1}^{M} \tau_{m,(k,j)} = 1, \forall (k,j),
\] (2.9)
2.3 Resource Allocation Schemes

where

\[
L(p, q, \pi, \tau, \nu, \lambda) = \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau_{m,(k,j)} \log_2 \left( 1 + \frac{a_{m,k} p_{m,k} b_{j} q_{j}}{1 + a_{m,k} p_{m,k} + b_{j} q_{j}} \right) \\
+ \sum_{m=1}^{M} \nu_{m} \left( P_{m} - \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau_{m,(k,j)} p_{m,k} \right) + \lambda \left( Q - \sum_{j=1}^{K} q_{j} \right),
\]

(2.10)

and \( \lambda, \nu = [\nu_1, \ldots, \nu_M]^T \) are the associated dual variables.

The dual problem is then defined as

\[
\min_{\nu \geq 0, \lambda \geq 0} D(\nu, \lambda).
\]

(2.11)

2.3.2 Joint Resource Allocation Scheme

Before tackling the dual problem (2.11), we need to first find the dual function (2.9) for given initial \( \lambda \) and \( \nu \). The dual function can be rewritten as

\[
D(\nu, \lambda) = \max_{p, q, \pi, \tau} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau_{m,(k,j)} \left( \frac{r_{m,(k,j)} - \lambda q_{j}}{\epsilon_{m,(k,j)}} \right) + \sum_{m=1}^{M} \nu_{m} P_{m} + \lambda Q
\]

(2.12)

s.t. \( \sum_{k=1}^{K} \pi_{k,j} = 1, \forall j, \sum_{j=1}^{K} \pi_{k,j} = 1, \forall k, \sum_{m=1}^{M} \tau_{m,(k,j)} = 1, \forall (k, j), \)

where \( \epsilon_{m,(k,j)} \) is defined as the corresponding item. For given \( \pi, \tau \), the optimal \( p \) and \( q \) could be found from

\[
D(\nu, \lambda) = \max_{p, q} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \epsilon_{m,(k,j)} + \sum_{m=1}^{M} \nu_{m} P_{m} + \lambda Q,
\]

(2.13)

which can be further decomposed into the following \( MK^2 \) sub-problems:

\[
\max_{p_{m,k} \geq 0, q_{j} \geq 0} \epsilon_{m,(k,j)}, \quad \forall m, (k, j).
\]

(2.14)

Although \( \epsilon_{m,(k,j)} \) in (3.10) is not jointly concave in \( p_{m,k} \) and \( q_{j} \), a commonly adopted technique is to apply the high SNR approximation [37] and replace
2.3 Resource Allocation Schemes

\[ \log_2 \left(1 + \frac{a_{m,k} p_{m,k} b_j q_j}{1 + a_{m,k} p_{m,k} + b_j q_j} \right) \]  
with concave function \( \log_2 \left(1 + \frac{a_{m,k} p_{m,k} b_j q_j}{a_{m,k} p_{m,k} + b_j q_j} \right) \). Thus, (3.10) becomes

\[ \max_{p_{m,k}, q_j} \log_2 \left(1 + \frac{a_{m,k} p_{m,k} b_j q_j}{a_{m,k} p_{m,k} + b_j q_j} \right) - \nu_m p_{m,k} - \lambda q_j \quad (2.15) \]

s.t. \( p_{m,k} \geq 0, \ q_j \geq 0, \) for all \( m, k, j \). In particular, [38] demonstrated that the resource allocation obtained from (2.15) yields very close results to the actual throughput even in the low SNR scenario. Since the above problem is now in the convex form, we refer to KKT conditions and first define the Lagrangian \( J \) associated with (2.15) such that

\[ J = \log_2 \left(1 + \frac{a_{m,k} p_{m,k} b_j q_j}{a_{m,k} p_{m,k} + b_j q_j} \right) + (\alpha_{m,k} - \nu_m) p_{m,k} + (\beta_j - \lambda) q_j. \quad (2.16) \]

The KKT conditions can be written as follows

\[ p_{m,k} \geq 0 \]
\[ \alpha_{m,k} \geq 0 \]
\[ \alpha_{m,k} p_{m,k} = 0 \]

\[ \frac{\partial}{\partial p_{m,k}} \left( \log_2 \left(1 + \frac{a_{m,k} p_{m,k} b_j q_j}{a_{m,k} p_{m,k} + b_j q_j} \right) + (\alpha_{m,k} - \nu_m) p_{m,k} + (\beta_j - \lambda) q_j \right) = 0 \quad (2.17) \]
\[ q_j \geq 0 \]
\[ \beta_j \geq 0 \]
\[ \beta_j q_j = 0 \]

\[ \frac{\partial}{\partial q_j} \left( \log_2 \left(1 + \frac{a_{m,k} p_{m,k} b_j q_j}{a_{m,k} p_{m,k} + b_j q_j} \right) + (\alpha_{m,k} - \nu_m) p_{m,k} + (\beta_j - \lambda) q_j \right) = 0. \]

From the fourth condition, we get

\[ \alpha_{m,k} = \nu_m - \frac{a_{m,k} q_j^2 b_j^2}{(p_{m,k} a_{m,k} + q_j b_j) (p_{m,k} a_{m,k} + q_j b_j + p_{m,k} a_{m,k} q_j b_j)}. \quad (2.18) \]

From \( \alpha_{m,k} \geq 0 \), we obtain

\[ \nu_m \geq \frac{a_{m,k} q_j^2 b_j^2}{(q_j b_j)^2 + p_{m,k} (p_{m,k} a_{m,k}^2 + a_{m,k} q_j b_j (p_{m,k} a_{m,k} + q_j b_j + 2))}. \quad (2.19) \]
2.3 Resource Allocation Schemes

With \(\alpha_{m,k}p_{m,k} = 0\), we have

\[
p_{m,k} \left( \nu_m - \frac{a_{m,k}q_j^2}{(p_{m,k}a_{m,k} + q_j b_j)(p_{m,k}a_{m,k} + q_j b_j + p_{m,k}a_{m,k}q_j b_j)} \right) = 0. \tag{2.20}
\]

(a) If \(\nu_m < a_{m,k}\), then the condition (2.19) can only be fulfilled when \(p_{m,k} > 0\). In this case (2.20) holds only if

\[
\nu_m = \frac{a_{m,k}(q_j)^2}{(p_{m,k}a_{m,k} + q_j b_j)(p_{m,k}a_{m,k} + q_j b_j + p_{m,k}a_{m,k}q_j b_j)}. \tag{2.21}
\]

(b) If \(\nu_m \geq a_{m,k}\), with \(p_{m,k} > 0\) it is impossible to meet (2.20). This implies that \(p_{m,k} = 0\).

From the last condition in (2.17), we get

\[
\beta_j = \lambda - \frac{b_j p_{m,k}^2}{(p_{m,k}a_{m,k} + q_j b_j)(p_{m,k}a_{m,k} + q_j b_j + p_{m,k}a_{m,k}q_j b_j)}. \tag{2.22}
\]

The Lagrangian in (2.16) is symmetric with respect to the \(p_{m,k}\) and \(q_j\). Therefore, for \(q_j > 0\) we obtain an expression similar to (2.21),

\[
\lambda = -\frac{b_j p_{m,k}^2}{(p_{m,k}a_{m,k} + q_j b_j)(p_{m,k}a_{m,k} + q_j b_j + p_{m,k}a_{m,k}q_j b_j)}. \tag{2.23}
\]

Otherwise \(q_j = 0\).

Now there are four possible cases:

1: \(p_{m,k} = 0, q_j > 0\): Substituting \(p_{m,k} = 0\) into (2.23), we get \(q_j = 0\). So both \(p_{m,k}\) and \(q_j\) are zero.

2: \(p_{m,k} > 0, q_j = 0\): Substituting \(q_j = 0\) into (2.21) implies that \(p_{m,k} = 0\). Therefore, again the power over both hops is zero.

3: \(p_{m,k} = 0, q_j = 0\).

4: \(p_{m,k} > 0, q_j > 0\): Solving equation (2.21) we obtain the optimal value of \(p_{m,k}\) as shown in the following equation

\[
p_{m,k}^* = \frac{1}{\nu_m \lambda} \left( 1 + \sqrt{\frac{\lambda}{\nu_m a_{m,k} b_j}} \right) \left( 1 - \frac{\sqrt{\frac{\lambda a_{m,k}}{\nu_m b_j} + \sqrt{\frac{\lambda a_{m,k}}{\nu_m b_j}}}^2}{a_{m,k} b_j} \right)^+, \tag{2.24}
\]
2.3 Resource Allocation Schemes

and similarly the solution of (2.23) yields the optimal value of \( q_j \) as

\[
q_j^* = \frac{1}{\lambda_m \left( 1 + \sqrt{\frac{r_{by}}{\lambda_m a_{m,k}}} \right)} \left( \frac{1}{\nu_m} - \left( \sqrt{\nu_j} + \sqrt{\frac{\lambda}{\nu_m a_{m,k}}} \right)^2 \right) ,
\]

(2.25)

where \( (x)^+ \triangleq \max \{0, x\} \).

The above discussion shows that there are only two possible case, i.e., either both \( p_{m,k} \) and \( q_j \) are positive or zero. In other words, if power allocated to a particular sub-carrier over either hop is zero, no power is allocated to its corresponding sub-carrier over the other hop.

Substituting (2.24) and (2.25) into (2.12), we obtain

\[
D(\nu, \lambda) = \max_{\pi, \tau} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau_{m,(k,j)} F_{m,(k,j)}(\nu, \lambda) + \sum_{m=1}^{M} \nu_m P_m + \lambda Q \quad (2.26)
\]

s.t.

\[
\sum_{k=1}^{K} \pi_{k,j} = 1, \forall j, \quad \sum_{j=1}^{K} \pi_{k,j} = 1, \forall k, \quad \sum_{m=1}^{M} \tau_{m,(k,j)} = 1, \quad \forall (k, j),
\]

where the function \( F_{m,(k,j)}(\nu, \lambda) \) is obtained by substituting \( p_{m,k}^* \) and \( q_j^* \) into \( e_{m,(k,j)} \).

Next, we look into the optimum sub-carrier allocation under a given sub-carrier pairing \((k, j)\). The dual function (2.26) can be written as

\[
D(\nu, \lambda) = \max_{\tau} \sum_{m=1}^{M} \tau_{m,(k,j)} F_{m,(k,j)}(\nu, \lambda) + \sum_{m=1}^{M} \nu_m P_m + \lambda Q \quad (2.27)
\]

s.t.

\[
\sum_{m=1}^{M} \tau_{m,(k,j)} = 1, \quad \forall (k, j).
\]

The optimal solution is simply choosing a user that has the maximum value of \( F_{m,(k,j)}(\nu, \lambda) \). Let the user index be \( m^* \), i.e.,

\[
m^* = \arg \max_m F_{m,(k,j)}(\nu, \lambda), \quad \forall (k, j).
\]

(2.28)

Then

\[
\tau_{m,(k,j)}^* = \begin{cases} 
1, & \text{for } m = m^* \\
0, & \text{otherwise}.
\end{cases}
\]

(2.29)
2.3 Resource Allocation Schemes

It remains to find the optimal sub-carrier pairing $\pi$. Substituting (2.29) into (2.26), we obtain

$$D(\nu, \lambda) = \max_{\pi} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} F_{m^*,(k,j)}(\nu, \lambda) + \sum_{m=1}^{M} \nu_{m} P_{m} + \lambda Q$$

\hspace{1cm} \text{(2.30)}

s.t. \hspace{1cm} \sum_{k=1}^{K} \pi_{k,j} = 1, \forall j, \sum_{j=1}^{K} \pi_{k,j} = 1, \forall k. \hspace{1cm} \text{(2.31)}

Let $F$ be a $K \times K$ matrix such that

$$\begin{bmatrix}
F_{m^*,(1,1)} & F_{m^*,(1,2)} & \cdots & F_{m^*,(1,K)} \\
F_{m^*,(2,1)} & F_{m^*,(2,2)} & \cdots & F_{m^*,(2,K)} \\
\vdots & \vdots & \ddots & \vdots \\
F_{m^*,(K-1,1)} & F_{m^*,(K-1,2)} & \cdots & F_{m^*,(K-1,K)} \\
F_{m^*,(K,1)} & F_{m^*,(K,2)} & \cdots & F_{m^*,(K,K)}
\end{bmatrix}$$

\hspace{1cm} \text{(2.32)}

The matrix $F$ can be considered as a profit matrix with row indices being different customers and column indices being different items to be sold. The value of each entry can be treated as the profit by selling a particular item to a particular customer. Problem (2.30) is equivalent to maximizing the sum profit by choosing the best selling strategy that can only sell one item to one customer. Such kind of linear assignment problem can be solved efficiently from the standard Hungarian algorithm with time complexity of $O(K^3)$ [39]. The steps of Hungarian algorithm are briefly described as follows [39]:

1) Subtract the values in each row from the maximum number in the row.

2) Subtract the minimum number in each column from the entire column.

3) Cover all zeroes in the matrix with as few lines (horizontal and/or vertical) as possible.

4) If the number of lines equals the size of the matrix, find the solution. If the number of lines is less than the size of the matrix, find the minimum number
2.3 Resource Allocation Schemes

that is uncovered. Subtract it from all uncovered values and add it to values at the intersections of lines.

5) Repeat step 3 to step 4 until there is a solution.

Denote the optimal pairing found from Hungarian algorithm as \( \pi_{k,j}^* \). Substituting \( \pi_{k,j}^* \), \( \tau_{m,(k,j)}^* \), \( p_{m,k}^* \), and \( q_j^* \) into (2.9) gives the dual function \( D(\lambda, \nu) \).

Next we need to find the optimal dual variables from the dual problem (2.11). From the sub-gradient method [40], we could pick up initial dual variables \( \nu^{(0)} \) and \( \lambda^{(0)} \). Then the dual variables at \((i+1)\)-th iteration could be updated as

\[
\nu_m^{(i+1)} = \left( \nu_m^{(i)} + \delta^{(i)} \Delta^{(i)}(\nu_m) \right)^+, \forall m, \tag{2.33}
\]

\[
\lambda^{(i+1)} = \left( \lambda^{(i)} + \delta^{(i)} \Delta^{(i)}(\lambda) \right)^+, \tag{2.34}
\]

where \( \delta^{(i)} \) is appropriate step size of the \( i \)th iteration. Moreover \( \Delta(\lambda) \), and \( \Delta(\nu_m) \) are the subgradients of \( D(\nu, \lambda) \) whose expressions can be found through the following proposition:

Proposition 1: The subgradients of \( D(\nu, \lambda) \) at \( \nu \) and \( \lambda \) are given by, respectively,

\[
\Delta(\nu_m) = P_m - \sum_{k=1}^K \sum_{j=1}^K \pi_{k,j} \tau_{m,(k,j)} p_{m,k}^*, \quad \forall m, \tag{2.35}
\]

\[
\Delta(\lambda) = Q - \sum_{j=1}^K q_j^*, \tag{2.36}
\]

where \( p_{m,k}^* \) and \( q_j^* \) are the optimal power values obtained from (2.24) and (2.25) at \( \nu \) and \( \lambda \).

Proof: For the given \((\pi, \tau, \nu, \lambda)\), the dual function in (2.9) can be re-stated as

\[
D(\nu, \lambda) = \max_{p,q} \sum_{k=1}^K \sum_{j=1}^K \pi_{k,j} \log_2 \left( 1 + \frac{(\sqrt{p_{m,k} h_k} \sqrt{q_j} g_j)^2}{\sigma_j^2 (\sqrt{p_{m,k} h_k})^2 + \sigma_k^2 (\sqrt{q_j} g_j)^2} \right) - \nu_m p_{m,k}
\]

\[
+ \lambda \left( Q - \sum_{j=1}^K q_j \right) + \sum_{m=1}^M \nu_m P_m. \tag{2.37}
\]

\(^1\)Note that, for each iteration, all the variables \( \pi_{k,j}^* \), \( \tau_{m,(k,j)}^* \), \( p_{m,k}^* \), \( q_j^* \), \( \forall m,k,j \) should be re-computed under \( \lambda^{(i)} \) and \( \nu^{(i)} \). The iteration will be stopped once certain criterion is fulfilled.
2.3 Resource Allocation Schemes

Let \( D(\nu, \lambda) \) be the dual function for some \( \lambda \), i.e.

\[
D(\nu, \lambda) = \max_{p,q} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi(k,j) \left( \log_2 \left( 1 + \frac{(\sqrt{p_{m,k}} | h_k | \sqrt{q_j} | g_j |)^2}{\sigma_j^2 (\sqrt{p_{m,k}} | h_k |)^2 + \sigma_k^2 (\sqrt{q_j} | g_j |)^2} \right) - \nu_m p_{m,k} \right)
\]

\[+ \lambda \left( Q - \sum_{j=1}^{K} q_j^* \right) + \sum_{m=1}^{M} \nu_m P_m. \tag{2.38} \]

Let \( p_{m,k}^* \), \( q_j^* \) be the solution to problem (2.37) and \( \overline{p}_{m,k} \), \( \overline{q}_j \) be the solution to problem (2.38), then

\[
D(\nu, \lambda) = \sum_{k=1}^{K} \sum_{j=1}^{K} \pi(k,j) \left( \log_2 \left( 1 + \frac{(\sqrt{p_{m,k}} | h_k | \sqrt{q_j^*} | g_j |)^2}{\sigma_j^2 (\sqrt{p_{m,k}} | h_k |)^2 + \sigma_k^2 (\sqrt{q_j^*} | g_j |)^2} \right) - \nu_m p_{m,k}^* \right)
\]

\[+ \lambda \left( Q - \sum_{j=1}^{K} q_j^* \right) + \sum_{m=1}^{M} \nu_m P_m
\]

\[\geq \sum_{k=1}^{K} \sum_{j=1}^{K} \pi(k,j) \left( \log_2 \left( 1 + \frac{(\sqrt{\overline{p}_{m,k}} | h_k | \sqrt{q_j^*} | g_j |)^2}{\sigma_j^2 (\sqrt{\overline{p}_{m,k}} | h_k |)^2 + \sigma_k^2 (\sqrt{q_j^*} | g_j |)^2} \right) - \nu_m p_{m,k}^* \right)
\]

\[+ \lambda \left( Q - \sum_{j=1}^{K} q_j^* \right) + \lambda (\lambda - \lambda) \left( Q - \sum_{j=1}^{K} q_j^* \right) + \sum_{m=1}^{M} \nu_m P_m
\]

\[= D(\nu, \lambda) + (\lambda - \lambda) \left( Q - \sum_{j=1}^{K} q_j^* \right). \]

Since the subgradient of a function \( f \) at \( x \) is \( c \) that satisfies the inequality \( f(y) \geq f(x) + c(y - x) \) for any \( y \) \([40]\), \( \left( Q - \sum_{j=1}^{K} q_j^* \right) \) is exactly the subgradient \( \Delta(\lambda) \). The subgradient \( \Delta(\nu_m) \) can be proved in a similar way.

The optimal power allocation has complexity of \( O(MK^2) \) and each maximization operation in (2.29) requires a complexity of \( O(M) \). If the sub-gradient method requires \( I \) iterations to converge, the total complexity of the proposed scheme becomes \( O(1K^2(2M + K)) \).
2.3 Resource Allocation Schemes

2.3.3 Proposed Low-Complexity Suboptimal Solution

The algorithm discussed in previous subsection gives a near optimal solution when the number of sub-carriers is large. However the computational complexity also increases with the increasing of the number of sub-carriers and the number of users. Here we provide a suboptimal approach which trades the performance for complexity. We divide the optimization (2.8) into two separate sub-problems as follows:

1. Fix \( p_{m;k}, q_j \), and optimize \( C \) over \( \pi_{k,j} \) and \( \tau_{m,(k,j)} \).

2. Fix \( \pi_{k,j}, \tau_{m,(k,j)} \), and optimize \( C \) by varying \( p_{m;k} \) and \( q_j \).

We also propose that each sub-carrier of the first hop will be allocated to a user regardless of the pairing strategy in the second hop. Therefore we will replace the notation \( \tau_{m,(k,j)} \) by \( \tau_{m,k} \), indicating that the sub-carrier allocation is not related with the index \( j \).

The complete sub-optimal algorithm can be outlined as

1) Define: \( U = \{1, \ldots, M\}, \: B = \{1, \ldots, K\}, \: C = \{1, \ldots, K\}, \: A_m = \{h_{m,1}, \ldots, h_{m,K}\}, \: A = \{A_1, \ldots, A_M\}, \: G = \{g_1, \ldots, g_K\}, \: Q = \{q_1, \ldots, q_K\}, \: H = \emptyset, \: P = \emptyset, \: \tau = \emptyset, \: \pi = \emptyset, \: V = \emptyset, \: p_{m,k} = \frac{P_m}{K}, \: q_j = \frac{Q}{K}, \: \tau_{m,k} = 0 \) for all \( m \in U \), \( k \in B \) and \( j \in C \).

2) for \( k = 1 \) to \( K \)

a) Find \( m \) s.t. \( p_{m,k}|h_{m,k}|^2 \geq p_{x,k}|h_{x,k}|^2 \) for all \( x \in U, h_{m,k} \in A, \) and \( h_{x,k} \in A \)

b) Set \( \tau_{m,k} = 1, \: \tau = \tau \cup \{\tau_{m,k}\}, \: H = H \cup \{h_{m,k}\}, \) and \( P = P \cup \{p_{m,k}\} \)

3) Set \( V = H. \) while \( V \neq \emptyset, \) do

a) Find \( k \) s.t. \( P(k)V(k) \geq P(y)V(y) \) for all \( k, y \in B \)

b) Find \( j \) s.t. \( Q(j)G(j) \geq Q(z)G(z) \) for all \( j, z \in C \)

c) Set \( \pi_{k,j} = 1, \: \pi = \pi \cup \{\pi_{k,j}\}, \: V = V - \{V(k)\}, \: B = B - \{k\}, \: C = C - \{j\} \).

4) For the obtained \( \tau \) and \( \pi \), re-allocate the powers \( p_{m,k} \) and \( q_j \) for all \( m \in U, \: k \in B \) and \( j \in C \) using expressions (2.24) and (2.25).
2.4 Generalization to Multiple Relay Scenario

Here, $U$, $B$ and $C$ denote the sets of users, sub-carriers on MU-to-RS link, and sub-carriers on RS-to-BS link, respectively, and $\chi(t)$ represents the $t$-th element of the set $\chi$. In the first step, both MUs and RS equally distribute the available power to $K$ sub-carriers, such that $p_{m,k} = \frac{P}{K}, \forall m$ and $q_j = \frac{Q}{K}$. In the second step, sub-carrier allocation among different users is found by selecting $K$ best sub-carriers among all $MK$ sub-carriers, such that each sub-carrier is allocated to a unique user that has the best channel gain for this particular sub-carrier. This requires a complexity of $O(MK)$. Sub-carrier pairing is done in step 3, where the best sub-carrier on MU-to-RS link is paired with the best sub-carrier on RS-to-BS link, with total complexity of $O(2K)$. For $\tau$ and $\pi$ found from step 2 and step 3, we can derive the power allocation in step 4. The dual variables are found from sub-gradient method.

If the solution converges after $I'$ sub-gradient updates, the total complexity of suboptimal scheme becomes $O(K(I' + M + 2))$. Considering $I'$ close to $I$, because both are iterations for gradient type search, we can see that the complexity of suboptimal approach is much less than that of joint resource allocation scheme, where later in the simulations it can be observed that the suboptimal algorithm yields closed performance to the joint resource allocation scheme.

2.4 Generalization to Multiple Relay Scenario

In the previous section, we developed the resource allocation algorithm for multi-user single RS scenario. In this section, we extend our previous results to the uplink scenario with $N$ RSs. The multi-relay uplink system model is shown in Fig. 2.2.

The protocol is described as follows: MUs transmit the signals based on OFDMA over different sub-carriers to more than one RS, whereas a particular RS can serve more than one MUs. To avoid interference, an OFDMA transmission is adopted on the second hop such that different relays will only forward the received signals over different sub-carriers.

Let $h_{m,n,k}$, $p_{m,n,k}$, and $\sigma^2_{m,n,k}$ denote the channel coefficient, power allocation
2.4 Generalization to Multiple Relay Scenario

and the variance of the additive noise over the \( k \)-th sub-carrier between the \( m \)-th MU and the \( n \)-th RS. Moreover denote \( g_{n,j} \) as the channel coefficient of sub-carrier \( j \) between the \( n \)-th RS and BS. Based on the proposed protocol, one sub-carrier pair \((k, j)\) can only be assigned to one MU-RS pair \((m, n)\). The corresponding data throughput is given by

\[
\tau_{(m,n),(k,j)} = \frac{1}{2} \log_2 \left( 1 + \frac{p_{m,n,k} a_{m,n,k} q_{n,j} b_{n,j}}{1 + p_{m,n,k} a_{m,n,k} + q_{n,j} b_{n,j}} \right),
\]  

(2.39)

where \( a_{m,n,k} = \frac{h_{m,n,k}}{\sigma_{m,n,k}^2}, b_{n,j} = \frac{g_{n,j}}{\sigma_{n,j}^2}, \) and \( q_{n,j}, \sigma_{n,j}^2 \) are the power allocation and the variance of the additive noise over the \( j \)-th sub-carrier during the second hop, respectively. Let \( \tau_{(m,n),(k,j)} \) be a binary variable indicating the sub-carrier pair allocation, i.e., \( \tau_{(m,n),(k,j)} = 1 \) if sub-carrier pair \((k, j)\) is allocated to MU-RS pair \((m, n)\) and it must satisfy

\[
\sum_{n=1}^{N} \sum_{m=1}^{M} \tau_{(m,n),(k,j)} = 1, \quad \forall (k, j).
\]  

(2.40)

Meanwhile, each node has limited transmission power, i.e.,

\[
\sum_{n=1}^{N} \sum_{k=1}^{K} p_{m,n,k} \leq P_m, \forall m, \quad \sum_{j=1}^{K} q_{n,j} \leq Q_n, \forall n,
\]  

(2.41)
2.4 Generalization to Multiple Relay Scenario

where $Q_n$ is the total power of the $n$-th RS.

With a little bit abuse of notations, we re-define $\tau = \{\tau(m,n),(k,j)\}$, $p = \{p_{m,n,k}\}$, $q = \{q_{n,j}\}$. Then the optimization can be formulated as

$$
\max_{\tau,\pi,p,q} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau(m,n),(k,j) T(m,n),(k,j) 
$$

s.t. (2.31), (2.40), (2.41).

2.4.1 Joint Optimization

We solve problem (2.42) using the similar techniques in Section 2.3. The dual function is

$$
\bar{D}(\nu, \lambda_n) = \max_{\pi,\tau,p,q} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau(m,n),(k,j) e(m,n),(k,j) + \sum_{m=1}^{M} \nu_m P_m + \sum_{n=1}^{N} \lambda_n Q_n
$$

s.t. (2.31), (2.40), (2.41),

where $\nu = [\nu_1, \ldots, \nu_M]^T$ and $\lambda = [\lambda_1, \ldots, \lambda_N]^T$ are the dual variables, and

$$
e(m,n),(k,j) = \tau(m,n),(k,j) - \nu_m p_{m,n,k} - \lambda_n q_{n,j}. \quad (2.44)
$$

For a given $\pi, \tau$ and under the high SNR approximation, (3.9) can be decomposed into following $MNK^2$ sub-problems

$$
\max_{p_{m,n,k} \geq 0, q_{n,j} \geq 0} \log_2 \left( 1 + \frac{p_{m,n,k} a_{m,n,k} q_{n,j} b_{n,j}}{p_{m,n,k} a_{m,n,k} + q_{n,j} b_{n,j}} \right) - (\nu_m p_{m,n,k} + \lambda_n q_{n,j}), \quad \forall m, n, k, j.
$$

(2.45)

Similar to the previous section, the KKT conditions yields the optimal $p_{m,n,k}$ and $q_{n,j}$ as

$$
p^*_{m,n,k} = \frac{1}{\nu_m \sqrt{\lambda_n}} \left( 1 + \sqrt{\frac{\lambda_n a_{m,n,k}}{\nu_m b_{n,j}}} \right) \left( 1 - \frac{(\sqrt{a_{m,n,k}} + \sqrt{\frac{\nu_m}{\lambda_n} b_{n,j}})^2}{\lambda_n a_{m,n,k} b_{n,j}} \right). \quad (2.46)
$$

$$
q^*_{n,j} = \frac{1}{\nu_m \sqrt{\lambda_n}} \left( 1 + \sqrt{\frac{\nu_m b_{n,j}}{\lambda_n a_{m,n,k}}} \right) \left( 1 - \frac{(\sqrt{b_{n,j}} + \sqrt{\frac{\lambda_n}{\nu_m} a_{m,n,k}})^2}{\nu_m a_{m,n,k} b_{n,j}} \right). \quad (2.47)
$$
2.4 Generalization to Multiple Relay Scenario

The optimal power allocation (solving (2.45) for all \(m, n, k, j\)) has complexity of \(O(MNK^2)\). Substituting \(p_{m,n,k}^*\) and \(q_{n,j}^*\) into (3.9), we obtain

\[
\bar{D}(\nu, \lambda) = \max_{\pi, \tau} \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{k,j} \tau_{(m,n),(k,j)} F_{(m,n),(k,j)}(\nu, \lambda) + \sum_{m=1}^{M} \nu_{m} P_{m} + \sum_{n=1}^{N} \lambda_{n} Q_{n}
\]

\[\text{subject to (2.31), (2.40),} \]

where \(F_{(m,n),(k,j)}(\nu, \lambda)\) is obtained by substituting \(p_{m,n,k}^*\) and \(q_{n,j}^*\) into \(e_{(m,n),(k,j)}\).

For a given sub-carrier pairing, (3.11) becomes

\[
\bar{D}(\nu, \lambda) = \max_{\tau} \sum_{m=1}^{M} \sum_{n=1}^{N} \tau_{(m,n),(k,j)} F_{(m,n),(k,j)}(\nu, \lambda) + \sum_{m=1}^{M} \nu_{m} P_{m} + \sum_{n=1}^{N} \lambda_{n} Q_{n}
\]

\[\text{subject to (2.40).} \]

The optimal solution of (3.12) is obtained by choosing an MU-RS pair that maximizes \(F_{(m,n),(k,j)}(\nu, \lambda)\). Mathematically,

\[
\tau_{(m,n),(k,j)}^* = \begin{cases} 
1, & \text{for } ((m, n), (k, j)) = \arg \max_{(m,n)} F_{(m,n),(k,j)} \\
0, & \text{otherwise.} 
\end{cases}
\]

It then remains to find the optimal sub-carrier pairing \(\pi_{k,j}^*\), which can be similarly obtained from Hungarian method described in Section 2.3.

Finally, the dual function can be obtained by substituting \(\pi_{k,j}^*, \tau_{(m,n),(k,j)}^*, p_{m,n,k}^*\) and \(q_{n,j}^*\) into (3.9).

From the sub-gradient method, we iteratively update the dual variables as

\[
\nu_{m}^{i+1} = \left( \nu_{m}^{i} + \delta^{i}(P_{m} - \sum_{n=1}^{N} \sum_{k=1}^{K} p_{m,n,k}^*) \right)^+, \quad \forall m,
\]

\[
\lambda_{n}^{i+1} = \left( \lambda_{n}^{i} + \delta^{i}(Q_{n} - \sum_{j=1}^{K} q_{n,j}^*) \right)^+, \quad \forall n,
\]

and \(\pi_{k,j}^*, \tau_{(m,n),(k,j)}^*, p_{m,n,k}^*\) and \(q_{n,j}^*\) are updated accordingly.

The sub-carrier allocation operation in (2.50) requires a complexity of \(O(MN)\). If the sub-gradient method requires \(I''\) iterations to converge, the total complexity of the proposed scheme becomes \(O(I''(2MNK^2 + K^3))\).
2.4.2 Suboptimal Algorithm

To reduce the computational complexity, we extend the suboptimal algorithm in Section 2.3.3 to the multi-relay case. The idea is to allocate sub-carriers among RSs over the second hop in a similar fashion as for the MUs over the first hop.

We only illustrate the difference from the Section 2.3.3 here:

1) Select $K$ best sub-carriers at hop-1, such that each sub-carrier is allocated to a unique MU that has the best channel gain for this particular sub-carrier.

2) Select $K$ best sub-carriers at hop-2, where each sub-carrier is allocated to a unique RS that has the best channel gain for this sub-carrier.

3) Sort the $K$ selected sub-carriers from step 1 and the $K$ selected sub-carriers from step 2, respectively. Then pair them according to their magnitude.

4) For the sub-carrier allocation and pairing found in previous three step, find the power allocation following the same way in (2.46) and (2.47).

The algorithm requires a total complexity of $O(K(I'' + M + N + 2))$, where $I''$ denote the number of iterations required for sub gradient updates.

2.5 Simulation Results

In this section, we provide numerical examples to evaluate the performance of our proposed algorithms. The frequency domain channels are generated using i.i.d Rayleigh distributed time domain taps. A flat fading channel condition is assumed for each sub-carrier. We choose $K = 32$ and assign the same total power to all MUs and RSs. The figure of the merit is taken as the per tone rate, i.e., sum rate divided by $K$ and is obtained from the average of 10000 different channel realizations. Moreover, the rates are obtained from the actual rate expression (2.7) while the solutions are computed from the high SNR approximation. The following algorithms are compared
2.5 Simulation Results

- OptSol/JntSol: OptSol is the joint optimal solution proposed in subsection 2.3.2 and JntSol denote the joint optimal solution proposed in subsection 2.4.1.

- SubOpt: The suboptimal solution presented in subsection 2.3.3 or subsection 2.4.2.

- EP-ECarr: A solution without optimization where equal amount of sub-carriers are allocated among users and the available power is equally divided among the allocated sub-carriers.

- EP-RndCarr: In this case each user is randomly assigned an amount of sub-carriers and equal power distribution among the allocated sub-carriers is assumed.

We first present numerical results for uplink transmission with single relay capability. The noise variances are set as \( \sigma_{m,k}^2 = \sigma_j^2 \), \( \forall m,k,j \). The performance throughput of four different methods versus SNR for a system with \( M = 10 \) users are shown in the Fig. 2.3. We also display the objective of the dual problem at the solution points in the same figure. We first observe that the duality gap between the dual objective and OptSol is close to zero at all SNR region, which demonstrate the optimality of the proposed algorithm. This is consistent with the result presented from [35]. Moreover, it can be seen that OptSol exhibits best performance at all SNR values, e.g., it yields 1.1 dB and 2.6 dB gains over EP-ECarr and EP-RndCarr solutions, respectively, at the rate equal to 1 bits/s/Hz. In comparison, the rate loss of the proposed suboptimal solution is only 0.3 dB but it requires much less computational complexity.

Next we examine the performance of the end-to-end rate versus the number of MUs for a single relay system. The corresponding curves at SNR= 10 dB are shown in Fig. 2.4. It can be seen that OptSol always yields the best performance, while a significant gain over EP-ECarr and EP-RndCarr is observed when the number of the users increases. This is basically due to two facts: i) the performance loss without optimization will become significant; ii) no multi-user diversity is exploited.
2.5 Simulation Results

However, the gap between the SubOpt and OptSol is almost constant, which shows the robustness of the proposed sub-optimal algorithm. Interestingly, the average throughput increase with $M$ but tends to a constant value when $M$ is greater than 10. The threshold indicates the point when the single relay cannot support more multi-user diversity. The performance can be further improved if we apply more relays. This will be demonstrated in the later examples.

Now, we look into multiple relay scenario. To compare the results with the recent work [32], JSP-(with matching) shows the performance of the solution presented in Algorithm 3 [32], where sub-carrier assignment and power allocation problem is solved through primal decomposition approach. Further, a sub-carrier pairing policy is also developed in this algorithm.

The throughput performance of five different methods versus SNR for a system with $M = 10$ users and $N = 5$ are shown in Fig. 2.5. The objective value of the dual problem at the solution points is also displayed in the same figure. We first observe that the duality gap between the dual objective and JntSol is close to zero.

Figure 2.3: Rate versus SNR for $M = 10$ and $N = 1$. 
2.5 Simulation Results

Figure 2.4: Throughput versus the number of users for $N = 1$.

at all SNR region, which demonstrates the optimality of the proposed algorithm. This is consistent with the result presented in [35]. We also see that JntSol exhibits the best performance at all SNR values among all different methods. It yields 2.5 dB SNR gain over JSP-(with matching). The reason is that JSP-(with matching) does not exploit diversity from multiple relay stations and only utilizes a suboptimal pairing strategy. Moreover, JntSol achieves large performance gains over two trivial methods, i.e., EP-ECarr and EP-RndCarr methods. In comparison, the rate loss of the proposed SubOpt is only 0.6 dB but it requires less computational complexity.

Next we examine the performance of the end-to-end rate versus the number of MUs for $N = 5$ relay system and the corresponding curves at SNR= 10 dB are shown in Fig. 2.6. Similar to the previous example, we see that JntSol always yields the best performance among different methods. The gain of JntSol over EP-ECarr and EP-RndCarr becomes large when the number of the users increases due to two facts: i) the performance loss without optimization will become significant; ii) no multi-user diversity is exploited. However, the gap between the SubOpt and JntSol is
2.5 Simulation Results

![Graph showing throughput versus SNR for M = 10 and N = 5.](image)

Figure 2.5: Throughput versus SNR for $M = 10$ and $N = 5$.

almost constant, which shows the robustness of the proposed sub-optimal algorithm. Moreover, the JntSol also performs much better than JSP-(with matching) due to the same reasons.

To make the comparison complete, we show the throughput performance versus the number of relays in Fig. 2.7. The number of users is taken as $M = 6$ and SNR is set as 10 dB. The key observation is that the gap between JntSol and JSP-(with matching) increases when the number of relay becomes large. This is due to the fact that in JSP-(with matching) a user can use at most one RS so the difference from the proposed JntSol becomes large. Other observations are similar to those in Fig. 2.7.

It is then of interest to check the performance of the proposed method when simultaneously varying the number of both users and relays. The curves for $N = 1, 2, 3, 4$ relays versus $M$ are shown in Fig. 2.8 for SNR = 10 dB. We numerically draw the conclusion that in order to increase the data throughput, both the number of the users and the number of relays should be increased simultaneously. It is
2.6 Summary

In this chapter, we studied the resource allocation problem in relay aided uplink multi-user multi-carrier system. First, considering the multi-user single relay network, the optimization is formulated under a unified framework where the power allocation over the sub-carriers, the sub-carrier assignment to the users, and the sub-carrier pairing at the relay nodes are jointly optimized. The sum throughput of all users is maximized subject to the limited available power budgets of the user and the relay nodes. Exploiting dual techniques, the joint problem is decomposed into independent sub-problems such that

- For each valid sub-carrier pair, the power allocation is obtained from the KKT conditions.

also interesting to notice the similarity with the traditional MIMO system where the throughput could only be improved when the numbers of the antennas at both transceivers are increased.

Figure 2.6: Throughput versus the number of users for $N = 5$. 
2.6 Summary

Figure 2.7: Throughput versus the number of relays for $M = 6$.

- For a sub-carrier pair, the optimal throughput is obtained from the optimal power allocation. A sub-carrier pair is assigned to a user which has the maximum optimal throughput among all other candidates.

- A profit matrix is obtained for the assigned sub-carrier pairs. The optimal pairing is then obtained from the Hungarian method.

The dual problem is solved from the sub-gradient algorithm and a near optimal solution is obtained. In order to reduce the computational complexity, we also presented a suboptimal algorithm which separates the joint optimization into three steps such that: 1) The sub-carrier assignment is obtained under equal power distribution at each node. 2) The allocated sub-carriers are then paired in such a way that the sub-carrier with highest channel gain over the first hop is paired with the sub-carrier of best gain at the second hop. 3) For the obtained sub-carrier allocation and the pairing, the optimal power is obtained from the waterfilling. The suboptimal
Figure 2.8: Throughput versus the number of users and the number of relays for JntSol.

algorithm demonstrated its comparable performance via numerical examples. The developed algorithms are also extended to the multiple relay scenario where each relay imposes a separate power constraint. An OFDMA transmission is adopted at the relay nodes such that each sub-carrier pair is assigned to a unique user-relay pair. As expected, increasing the relay nodes enhances the system performance.
Chapter 3

Resource Optimization in Bidirectional Relay Networks

In this chapter, we study resource optimization in multi-user bidirectional relay networks. Based on the studies in previous chapter, we derive an asymptotically optimal solution for joint resource optimization and further design a low complexity algorithm to trade the performance for computational complexity. Finally, simulation results are provided to demonstrate the performance gain of the proposed algorithms.

3.1 Introduction

Overview

Most of the existing communication systems are bidirectional such that the roles of the source node and the destination node alternate from time to time. In a relay aided scenario, the bidirectional transmission can be simply achieved by applying twice the principle of relaying scheme from each direction. Let $U_1$ and $U_2$ be the two users who can communicate with each other through a relay station $R$. Due to half duplex nature of relay terminals, the traditional relaying transmission requires four phases for a complete bidirectional transmission round as shown in Fig. 3.1.
A new technology called TWRN has been developed in [41], [42]. The TWRN exploits the recently developed advanced coding techniques [43] to overcome the spectral efficiency loss caused by the half-duplex constraint in the one-way relay network (OWRN). Compared with the conventional one-way relay partner, two-way communication provides an improved spectral efficiency with a two phase protocol, where in the first phase, the two terminals simultaneously transmit their signals to the relay node, and in the second phase the relay transmits the received signals to both ends. A simple three-node TWRN is shown in Fig. 3.2, where one round of data communication is divided into two phases. During Phase $t_1$, signals are sent out simultaneously from both $U_1$ and $U_2$, and then are superimposed at the $R$. During Phase $t_2$, the relay broadcasts the received signals to both terminals. Since $U_i$'s $i = 1, 2$ know their transmitted signal in the first phase, they can subtract the self-signal component from the received signal and recover the desired information. Compared with four phases transmission, bidirectional relay transmission significantly enhances the transmission throughput [42], [43].

**Related Work**

Shannon first studied the two way relay channel in [44] and recently a number of two way relaying protocols for AF and DF schemes have been proposed in the literature [45]–[47]. Considering multiple relay nodes, distributed space time coding scheme and the opportunistic relaying scheme have been presented in [48] and [49].

Resource allocation techniques in OFDM based TWRNs have been proposed in [50]–[55]. The authors in [50] studied the throughput maximization problem in a
three node network, where two user terminals exchange information with the help of a relay node using OFDM transmission, subject to an individual power constraint at each node. The results showed an enhanced system performance from an optimized power allocation via dual decomposition technique and a greedy tone permutation scheme. This scheme is further exploited in [51] under a total power constraint where a two step power allocation strategy was proposed. Joint power allocation and sub-carrier assignment problem in multiple-relay scenario, where two terminals exchange information with the help of more than one intermediate relay nodes, was considered in [52]. The problem is solved by a suboptimal algorithm where each resource is optimized by fixing the other. The authors further applied the idea to the OFDMA based multi-user multi-relay systems in [53] and proposed a sub-carrier allocation algorithm for the known power allocation. The work in [54] studied the power and sub-carrier allocation problem in OFDMA multi-user relay network. More recently, relay power allocation problem in a multi-user system, where a number of user pairs exchange information through a single relay station, was considered in [55]. However a unified resource allocation scheme considering tone permutation, power optimization, and sub-carrier allocation all together has not been designed.

Motivation

The two phases of transmission in bidirectional relaying are generally categorized as following

- *Multiple-Access Phase* (MAP): In MAP the two end terminals simultaneously transmit information to the relay node.
3.1 Introduction

- **Broadcast Phase (BCP):** The relay processes the information according to the relaying mode (AF or DF) and broadcasts the message in BCP.

Conventionally, in OFDM based bidirectional communication, the signal received over a sub-carrier in MAP is re-transmitted at the same sub-carrier in BCP. Due to the independent nature of the channels in the two phases, a deep faded sub-carrier in MAP may have high SNR in BCP. Thus, using the same sub-carrier in MAP and BCP is clearly suboptimal and a better utilization of channel dynamics could be achieved with a careful tone-matching strategy.

The previous reported works [50]–[55] have shown the enhanced throughput results in OFDM systems by optimizing different units of resources separately. However, due to the nature of types of resources and the multi-user TWRN system model, the different resources are tightly coupled with each other. For example, a bad sub-carrier $k$ in MAP for a pre-assigned user pair ($U_1, U_2$) may be good for another user pair ($U_3, U_4$) and a reverse could exist in BCP. Thus, the sub-carriers should not only be carefully matched in the two phases (MAP and BCP) but also be allocated adaptively for different users. Further, the distributed nature of the wireless systems prohibits us to impose a total power constraint over all nodes. Hence, we assume that each node has a limited power supply, which makes our consideration closer to practical scenarios.

**Contributions**

The main contributions of this chapter are:

- Our target is to maximize the end-to-end system transmission rate of the OFDMA based multi-user bidirectional relay network subject to individual power budget constraint of each transmitting node. We present a new unified framework for jointly optimizing different types of resources, namely,
  
  - Power allocation at user and relay terminals such that each user allocates powers to a unique set of sub-carriers allocated to him in MAP and the
3.2 System Model

relay node distributes its power to all available OFDM sub-carriers in BCP.

- Sub-carrier assignment among different users such that the same sub-carrier should be assigned to the two terminals in a user-pair for a particular transmission phase.
- The tone matching at the relay node, where the signal received in MAP over one sub-carrier is re-transmitted on a different (or same) sub-carrier in BCP.

- To reduce the complexity, we further propose a suboptimal method that sacrifices very little on the performance as demonstrated by the numerical examples.
- A complexity comparison of the joint and the suboptimal methods is provided and the numerical examples show a better performance over the existing trivial methods.

The rest of this chapter is organized as follows. In Section 3.2, we present the system model. The joint resource allocation problem is formulated in Section 3.3. In Section 3.4, we develop the dual decomposition method. The low complexity suboptimal algorithm is presented in Section 3.5. Simulation results are presented in Section 3.6 and conclusions are given in Section 3.7.

3.2 System Model

We consider a two-way multi-user relay network that consists of $M$ pre-assigned pairs of MUs and one fixed RS, all equipped with only one antenna that cannot transmit and receive simultaneously, as shown in Fig. 3.3. Further, we assume that a direct communication link is missing between two users and all the communication is carried out through RS which operates in AF mode. In MAP, all MUs transmit information to RS simultaneously via non-overlapping carriers. In BCP, the RS
3.2 System Model

broadcasts the received signal after certain processing, for example power amplifying and carrier permutation. The two users of the $m$-th user pair, denoted as $A_m$ and $B_m$, transmit simultaneously on the same carriers, for example the $k$th carrier in MAP,\(^1\) while the received signal will be sent back over the $j$-th sub-carrier in BCP. Assigning which carrier to which user-pair, as well as the matching strategy $(k,j)$ will be optimized in this work.

Denote the channel coefficient from $A_m$ to RS as $h_{m;k}$, the one from $B_m$ to RS as $g_{m;k}$, the one from RS to $A_m$ as $\tilde{h}_{m;j}$, and the one from RS to $B_m$ as $\tilde{g}_{m;j}$\(^2\) Then the received signal at RS is

$$y_{RS}^k = \sqrt{p_{A_{m;k}}^k}h_{m;k}x_{A_{m;k}}^k + \sqrt{p_{B_{m;k}}^k}g_{m;k}x_{B_{m;k}}^k + w_{RS}^k; \quad (3.1)$$

where $x_{A_{m;k}}^k$ and $x_{B_{m;k}}^k$ are the information symbols to be exchanged, $p_{A_{m;k}}^k$ and $p_{B_{m;k}}^k$ are the corresponding powers over the $k$-th carrier, and $w_{RS}^k$ is the additive white Gaussian noise with variance $\sigma^2$.

If the power allocated at RS over sub-carrier $j$ is represented as $p_j^R$, then the

---

\(^1\)That is to say, $A_{m_1}$ and $B_{m_2}$ will not transmit on the same carrier during MAP.

\(^2\)By letting $\tilde{h}_{m;j} = h_{m;j}$ and $\tilde{g}_{m;j} = g_{m;j}$, the scenario here reduces to the reciprocal channels.
3.3 Problem Formulation

signals received at the $m$-th user pair can be written as

\begin{align}
  y_{m,j}^A &= \sqrt{p_j^R h_{m,j} \rho_j} \sqrt{p_{m,k}^A h_{m,k} x_{m,k}^A} + \sqrt{p_j^R h_{m,j} \rho_j} \sqrt{p_{m,k}^B g_{m,k} x_{m,k}^B} \\
  &\quad + w_{m,j}^A, \\  y_{m,j}^B &= \sqrt{p_j^R \rho_j g_{m,j} \sqrt{p_{m,k}^B g_{m,k} x_{m,k}^B}} + \sqrt{p_j^R \rho_j g_{m,j} \sqrt{p_{m,k}^A h_{m,k} x_{m,k}^A}} \\
  &\quad + w_{m,j}^B,
\end{align}

(3.2)

where $\rho_j \triangleq \frac{1}{\sqrt{p_{m,k}^A h_{m,k}|^2 + p_{m,k}^B g_{m,k}|^2 + \sigma^2}}$ is the scaling factor to keep the power constraint, while $w_{m,j}^A$ and $w_{m,j}^B$ are the received additive white Gaussian noises (AWGN) at $A_m$ and $B_m$, respectively, both with variance $\sigma^2$. Assuming a perfect self-interference cancellation, the corresponding SNRs can be written as

\begin{align}
  \text{SNR}_{m,j}^A &= \frac{p_j^R h_{m,j}^2 \rho_j^2 p_{m,k}^B |g_{m,k}|^2}{(p_j^R \rho_j^2 h_{m,j}^2 + 1) \sigma^2}, \\
  \text{SNR}_{m,j}^B &= \frac{p_j^R g_{m,j}^2 \rho_j^2 p_{m,k}^A |h_{m,k}|^2}{(p_j^R \rho_j^2 g_{m,j}^2 + 1) \sigma^2}.
\end{align}

(3.4, 3.5)

3.3 Problem Formulation

Due to the exclusive tone matching constraint, each sub-carrier in MAP can only be paired with one sub-carrier in BCP. We then define $\pi_{(k,j)} \in \{0,1\}$ as the binary variable for the sub-carrier pairing such that $\pi_{(k,j)} = 1$ if the $k$-th sub-carrier is paired with the $j$-th sub-carrier, while $\pi_{(k,j)} = 0$ otherwise. Further, we define binary variables $\tau_{m,(k,j)} \in \{0,1\}$, such that $\tau_{m,(k,j)} = 1$ if sub-carrier pair $(k,j)$ is allocated to the $m$-th MU pair while $\tau_{m,(k,j)} = 0$ otherwise.

We seek to jointly optimize the sub-carrier allocation, sub-carrier pairing, and the power allocation such that the overall system throughput is maximized under individual power constraints at MUs and RS. Let $P_{A_m}$, $P_{R}$, and $P_{B_m}$ denote the total available powers at $A_m$, RS, and $B_m$, respectively. The optimization can be
3.4 Joint Resource Allocation Scheme

formulated as

$$\max_{\pi, \tau, p^A, p^B, p^R} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \tau_{m,(k,j)} \left( \frac{1}{2} C(\text{SNR}^A_{m,j}) + \frac{1}{2} C(\text{SNR}^B_{m,j}) \right)$$

(3.6)

s.t.

$$\sum_{k=1}^{K} \pi_{(k,j)} = 1, \forall j, \sum_{j=1}^{K} \pi_{(k,j)} = 1, \forall k,$$

$$\sum_{m=1}^{M} \tau_{m,(k,j)} = 1, \forall (k,j), \sum_{j=1}^{K} p^R_j \leq P_R,$$

$$\sum_{k=1}^{K} p^A_{m,k} \leq P_{A_m}, \forall m, \sum_{k=1}^{K} p^B_{m,k} \leq P_{B_m}, \forall m,$$

$$p^A_{m,k} \geq 0, \ p^R_j \geq 0, \ p^B_{m,k} \geq 0, \ \forall m, k, j,$$

where $C(x) \triangleq \log_2(1 + x)$, and $\pi = \{\pi_{(k,j)}\}$, $\tau = \{\tau_{m,(k,j)}\}$, $p^A = \{p^A_{m,k}\}$, $p^B = \{p^B_{m,k}\}$, $p^R = \{p^R_j\}$ for all $m = \{1, ..., M\}$, $k = \{1, ..., K\}$, $j = \{1, ..., K\}$. The $\frac{1}{2}$ factor appears due to the two time slots used for a complete transmission.

The first and the second constraints are originated from the fact that each sub-carrier in MAP can be coupled with one and only one sub-carrier in BCP and vice versa. The third constraint ensures the exclusive allocation of the sub-carrier pair $(k, j)$ to the $m$-th user pair $(A_m, B_m)$ only. However more than one sub-carrier pairs can be allocated to a particular MU pair. Other constraints represent individual power constraint at each node.

3.4 Joint Resource Allocation Scheme

It is easily known that (3.6) is a mixed integer non-linear programming problem [34], and we can solve the dual problem instead of the original problem [35]. The dual problem associated with the primal problem (3.6) is defined as [13]

$$\min_{\nu, \lambda, \eta} D(\nu, \lambda, \eta)$$

(3.7)

s.t. $\nu_m \geq 0, \eta_m \geq 0, \forall m, \lambda \geq 0,$
3.4 Joint Resource Allocation Scheme

where \( D(\nu, \lambda, \eta) \) is the dual function given by

\[
D(\nu, \lambda, \eta) = \max_{\pi, \tau, p^A, p^R, p^B} \left\{ \sum_{m=1}^M \sum_{k=1}^K \sum_{j=1}^K \pi(k,j) \tau_{m,k}(j) \left( \frac{1}{2} C(\text{SNR}_{m,j}^A) + \frac{1}{2} C(\text{SNR}_{m,j}^B) \right) \right. \\
+ \sum_{m=1}^M \nu_m \left( P_{Am} - \sum_{k=1}^K p_{m,k}^A \right) + \sum_{m=1}^M \eta_m \left( P_{Bm} - \sum_{k=1}^K p_{m,k}^B \right) \\
+ \lambda \left( P_R - \sum_{j=1}^K p_j^R \right) \bigg| \sum_{k=1}^K \pi(k,j) = 1, \forall j, \sum_{j=1}^K \pi(k,j) = 1, \forall k, \\
\left. \sum_{m=1}^M \tau_{m,k}(j) = 1, \forall (k,j) \right\}, \tag{3.8}
\]

and \( \nu = [\nu_1, \ldots, \nu_M]^T \), \( \lambda, \eta = [\eta_1, \ldots, \eta_M]^T \) are the associated Lagrange multipliers or the dual variables.

3.4.1 Lagrange Dual Decomposition: Solving the Dual Function

To proceed with the dual problem (3.7), we need to first find the dual function (3.8) for given initial \( \lambda, \nu, \) and \( \eta \). The dual function can be re-expressed as

\[
D(\nu, \lambda, \eta) = \max_{\pi, \tau, p^A, p^R, p^B} \left\{ \sum_{m=1}^M \sum_{k=1}^K \pi(k,j) \tau_{m,k}(j) \left( \frac{1}{2} C(\text{SNR}_{m,j}^A) + \frac{1}{2} C(\text{SNR}_{m,j}^B) \right) \right. \\
- \nu_m p_{m,k}^A - \lambda p_j^R - \eta_m p_{m,k}^B \right. \\
+ \sum_{m=1}^M \nu_m P_{Am} + \lambda P_R \\
+ \sum_{m=1}^M \eta_m P_{Bm} \bigg| \sum_{k=1}^K \pi(k,j) = 1, \forall j, \sum_{j=1}^K \pi(k,j) = 1, \forall k, \\
\left. \sum_{n=1}^N \sum_{m=1}^M \tau_{m,k}(j) = 1, \forall (k,j) \right\}. \tag{3.9}
\]

Clearly, for given \( \pi, \tau \), the optimal \( p^A, p^R, \) and \( p^B \) could be found from the following sub-problems:

\[
\max_{p_{m,k}^A, p_j^R, p_{m,k}^B} \frac{1}{2} C(\text{SNR}_{m,j}^A) + \frac{1}{2} C(\text{SNR}_{m,j}^B) - \nu_m p_{m,k}^A - \lambda p_j^R - \eta_m p_{m,k}^B, \tag{3.10}
\]

s.t. \( p_{m,k}^A \geq 0, p_j^R \geq 0, p_{m,k}^B \geq 0. \)
3.4 Joint Resource Allocation Scheme

We solve (3.10) for all $m, k, j$, thus there are total $MK^2$ sub-problems. The power allocation problem in (3.10) is non-convex and finding the closed form solution is not trivial. Nevertheless, the optimal solution $(\hat{p}_{A}^{m,k}, \hat{p}_{R}^{j}, \hat{p}_{B}^{m,k})$ can be obtained through searching over $p_{m,k}^{A}, p_{j}^{R},$ and $p_{m,k}^{B}$, assuming that each takes discrete values [50], [51]. This approach requires $O(Z^3)$ computational complexity where $Z$ is the number of power levels that can be taken by each of $p_{A}^{m,k}, p_{R}^{j},$ and $p_{B}^{m,k}$. Therefore the total complexity of solving power allocation for all $m, (k, j)$ is $O(MK^2Z^3)$.

Substituting optimal power values $\hat{p}_{A}^{m,k}, \hat{p}_{R}^{j},$ and $\hat{p}_{m,k}^{B}$ into (3.9), we obtain

$$D(\nu, \lambda, \eta) = \max_{\pi, \tau} \left\{ \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi(k,j) \tau_{m,(k,j)} F_{m,(k,j)} + \sum_{m=1}^{M} \nu_{m} P_{A_{m}} + \lambda P_{R} + \sum_{m=1}^{M} \eta_{m} P_{B_{m}} \right\},$$

(3.11)

where $F_{m,(k,j)}$ is obtained by substituting $\hat{p}_{A}^{m,k}, \hat{p}_{R}^{j},$ and $\hat{p}_{m,k}^{B}$ into the objective $\frac{1}{2} C (\text{SNR}_{m,j}^{A}) + \frac{1}{2} C (\text{SNR}_{m,j}^{B}) - \nu_{m} p_{m,k}^{A} - \lambda p_{j}^{R} - \eta_{m} p_{m,k}^{B}$.

To find the optimum sub-carrier allocation under a given tone matching, (3.11) becomes

$$\max_{\tau} \left\{ \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{j=1}^{K} \tau_{m,(k,j)} F_{m,(k,j)} + \sum_{m=1}^{M} \nu_{m} P_{A_{m}} + \lambda P_{R} + \sum_{m=1}^{M} \eta_{m} P_{B_{m}} \right\},$$

(3.12)

The optimal solution of (3.12) is obtained by choosing an MU pair that maximizes $F_{m,(k,j)}$, i.e.,

$$\hat{\tau}_{m,(k,j)} = \begin{cases} 1, & \text{for } m = \arg \max_{m} F_{m,(k,j)}, \forall (k, j), \\ 0, & \text{otherwise}. \end{cases}$$

(3.13)

For a given $\pi(k,j)$, each maximization operation in (3.13) has the complexity of $O(M)$ and the total complexity of solving sub-carrier allocation problem thus is $O(MK^2)$. 

52
3.4 Joint Resource Allocation Scheme

It remains to find the optimal tone matching $\hat{\pi}$. Substituting (3.13) into (3.11), we obtain

$$D(\nu, \lambda, \eta) = \max_{\pi} \left\{ \sum_{k=1}^{K} \sum_{j=1}^{K} \pi(k,j) F^*(m,(k,j)) + \sum_{m=1}^{M} \nu_m P_A m + \lambda P_R + \sum_{m=1}^{M} \eta_m P_B m \left| \sum_{k=1}^{K} \pi(k,j) = 1, \forall j; \sum_{j=1}^{K} \pi(k,j) = 1, \forall k \right. \right\}, \tag{3.14}$$

where $F^*(m,(k,j)) = \max_m F_{m,(k,j)}, \forall (k,j)$. We define a $K \times K$ matrix $F$ with the $(k,j)$-th entry $[F]_{k,j} = F^*(m,(k,j))$. The matrix $F$ can be considered as a profit matrix with row indices being different operators and column indices being different machines to be operated, i.e., a total of $K$ different machines to be operated by $K$ different operators. The value of each entry can be treated as the profit from operating a particular machine by a particular operator. Problem (3.14) is equivalent to maximizing the sum profit by choosing the best strategy where each operator $(k)$ can operate only one machine $(j)$. Such kind of linear assignment problem can be solved efficiently from the standard Hungarian algorithm with the complexity $O(K^3)$ [39].

Finally, the dual function can be obtained by substituting $\hat{\pi}$, $\hat{\tau}$, $\hat{p}^A$, $\hat{p}^R$, and $\hat{p}^B$ into (3.8).

3.4.2 Solving the Dual Problem with Sub-gradient Method

Next we solve the dual problem (3.7) to find the optimal values of dual variables. From the sub-gradient method [40], we could pick up initial dual variables $\lambda^{(0)}$, $\nu^{(0)}$, and $\eta^{(0)}$ to find the power allocation in (3.10). Then with the obtained $\hat{p}^A$, $\hat{p}^R$, and $\hat{p}^B$, the dual variables at $(i + 1)$-th iteration should be updated as

$$\nu^{(i+1)}_m = \left[ \nu^{(i)}_m - \delta^{(i)} \left( P_A m - \sum_{k=1}^{K} \sum_{j=1}^{K} \hat{\pi}(k,j) \hat{r}_{m,(k,j)} \hat{p}^A_{m,k} \right) \right]^+, \tag{3.15}$$

$$\eta^{(i+1)}_m = \left[ \eta^{(i)}_m - \delta^{(i)} \left( P_B m - \sum_{k=1}^{K} \sum_{j=1}^{K} \hat{\pi}(k,j) \hat{r}_{m,(k,j)} \hat{p}^B_{m,k} \right) \right]^+, \tag{3.16}$$

53
for all $m$, and

$$
\lambda^{(i+1)} = \left[ \lambda^{(i)} - \delta^{(i)} \left( P_R - \sum_{j=1}^{K} \hat{p}_j^R \right) \right]^+, \tag{3.17}
$$

where $[x]^+ \triangleq \max(0, x)$, and $\delta^{(i)}$ is an appropriate step size of the $i$th iteration. Note that, for each iteration, all the variables $\hat{r}_{(k,j)}$, $\hat{r}_{m,(k,j)}$, $\hat{p}_m^A$, $\hat{p}_m^B$, and $\hat{p}_j^R$ should be re-computed under $\lambda^{(i)}$, $\nu_m^{(i)}$, and $\eta_m^{(i)}$. The iteration will be stopped once certain criterion is fulfilled. Then, we normalize $\hat{p}_A$, $\hat{p}_R$, and $\hat{p}_B$ so that the power constraint at each node is satisfied.

If the dual objective function $D(\nu, \lambda, \eta)$ is minimized within $N$ iterations, the total computational complexity of our proposed scheme becomes $O(NK^2(M(Z^3 + 1) + K))$ which is much less than that of solving problem by exhaustive search, i.e., $O(NM^KZ^3)$.

### 3.5 Step-Wise Low-Complexity Resource Allocation Scheme

The algorithm derived in previous subsections provides a near optimal solution for the large number of sub-carriers. However the computational efficiency decreases with the increasing of $K$ and $M$. In this subsection we propose a suboptimal algorithm which trades the performance for lower complexity. We solve the optimization (3.6) following a step-wise approach where each resource is optimized while fixing the others. The algorithm is outlined as:

#### 3.5.1 Sub-carrier Allocation for Given Power Allocation

Initially, we fix the power allocation by equally distributing the available powers at RS and each MU to the $K$ sub-carriers, i.e., $p_k^R = \frac{P_R}{K}, \forall k$, $p_m^A = \frac{P_m^A}{K}, \forall m,k$, and $p_m^B = \frac{P_m^B}{K}, \forall m,k$. Then each sub-carrier $k$ is assigned to an $m$-th user pair,
3.5 Step-Wise Low-Complexity Resource Allocation Scheme

denoted as $m_k^*$, such that

$$ m_k^* = \arg \max_m \left( \text{SNR}^A_{m,k} + \text{SNR}^B_{m,k} \right), \forall k, $$

(3.18)

where

$$ \text{SNR}^A_{m,k} = \frac{p^R_k |\tilde{h}_{m,k}|^2 \rho^2_k p^R_{m,k} |g_{m,k}|^2}{(p^R_k \rho^2_k |\tilde{h}_{m,k}|^2 + 1) \sigma^2}, $$

(3.19)

$$ \text{SNR}^B_{m,k} = \frac{p^R_k |\tilde{g}_{m,k}|^2 \rho^2_k p^A_{m,k} |h_{m,k}|^2}{(p^R_k \rho^2_k |\tilde{g}_{m,k}|^2 + 1) \sigma^2}. $$

(3.20)

In this process a set of $K_m$ number of sub-carriers, denoted as $\Omega_m$, is assigned to $m$-th MU pair such that $0 \leq K_m \leq K$, and $\sum_{m=1}^M K_m = K$.

For a given $k$, obtaining the optimum $m_k^*$ in (3.18) requires a complexity of $O(M)$, and thus the total computational complexity of sub-carrier allocation becomes $O(MK)$ which is $NK$ times less than that from (3.13).

3.5.2 Tone Matching for Given Power Allocation and Sub-carrier Allocation

To find the tone matching, we first re-distribute the power at each of the MU such that $p^A_{m,k} = \frac{p_{A,m}}{K_m}$ and $p^B_{m,k} = \frac{p_{B,m}}{K_m}$, $\forall m, k \in \Omega_m$. For the $m$-th user pair, we choose a carrier $k^*$ in MAP such that

$$ k^* = \arg \max_{k \in \Omega_m} p^A_{m,k} |h_{m,k}|^2 + p^B_{m,k} |g_{m,k}|^2, $$

(3.21)

and pair it with sub-carrier $j^*$ in BCP, where

$$ j^* = \arg \max_{j \in \Omega_m} \left( \text{SNR}^A_{m,j,k^*} + \text{SNR}^B_{m,j,k^*} \right), $$

(3.22)

with

$$ \text{SNR}^A_{m,j,k^*} = \frac{p^R_j |\tilde{h}_{m,j}|^2 \rho^2_j p^B_{m,k^*} |g_{m,k^*}|^2}{(p^R_j \rho^2_j |\tilde{h}_{m,j}|^2 + 1) \sigma^2}, $$

(3.23)
3.6 Simulation Results

and

\[
\text{SNR}^B_{m,j,k^*} = \frac{p_j^R R_j^R \beta_{m,j}^2 p_{m,k^*}^2 |h_{m,k^*}|^2}{(p_j^R R_j^R |\beta_{m,j}|^2 + 1) \sigma^2}.
\]

(3.24)

Each of the maximization in (3.21) and (3.22) have the complexity of \(O(K_m)\), and hence the sub-carrier paring for all \(M\) users require the complexity of \(O(\sum_{m=1}^{M} 2K_m) = O(2K)\).

3.5.3 Power Allocation for Given Sub-carrier Allocation and Tone Matching

For the obtained sub-carrier allocation and tone matching, we recalculate the power allocation using the dual decomposition approach, where the dual function can be decomposed into \(K\) sub-problems, each being similar to (3.10). The dual variables are found from sub-gradient method in subsection 3.4.2. The solution will converge after \(N'\) updates of (3.15), (3.16), and (3.17).

The power allocation requires a complexity of \(O(N'KZ^3)\), and thus the total complexity of the algorithm from step 1 to step 3 is \(O(K(M + N'Z^3 + 2))\). Without loss of generality, let \(N' = N\). The overall complexity of the proposed suboptimal algorithm is much less than \(O(NK^2(M(Z^3 + 1) + K))\), the complexity of the joint resource allocation scheme.

3.6 Simulation Results

In this section, we provide simulation results to evaluate the performance of our proposed algorithms. We consider 6-tap channels taken from i.i.d. Gaussian random variables for all links, while the total number of sub-carriers is set as 32. Without loss of generality, we assume equal power at each node. The figure of the merit is taken as the per tone rate, i.e., sum rate divided by \(K\).

We compare the following algorithms:

* JntOpt: The joint optimal solution proposed in subsection 3.4.
3.6 Simulation Results

- SubOpt: The suboptimal solution presented in subsection 3.5.
- SolWOP: A solution where power allocation and sub-carrier allocation is optimized but tone matching is not considered. Algorithm follows the steps for joint optimization algorithm in subsection 3.4 with \( k = j \) and omits the tone matching.
- Static: A fixed resource allocation solution where each user is randomly assigned an amount of sub-carriers and then the available power is distributed evenly among the allocated sub-carriers. The tone permutation is not considered.

The complexity involved in each algorithms is summarized in Table 3.1, where \( N'' \) denotes the number of iterations required for sub-gradient convergence in SolWOP algorithm. Further, the running time of different schemes for different number of users at \( \text{SNR}= 10 \) are also displayed.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Complexity</th>
<th>Running Time (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JointOpt</td>
<td>( O(NK^2(M(Z^3 + 1) + K)) )</td>
<td>M=2 113.2310 M=5 297.6780 M=10 598.6954 M=15 868.5970 M=20 1177.769</td>
</tr>
<tr>
<td>SubOpt</td>
<td>( O(K(M + N''Z^3 + 2)) )</td>
<td>M=2 1.7810 M=5 1.8281 M=10 1.8910 M=15 1.8984 M=20 1.9010</td>
</tr>
<tr>
<td>SolWOP</td>
<td>( O(MN''K^2(Z^3 + 1)) )</td>
<td>M=2 3.6250 M=5 9.1240 M=10 19.0121 M=15 27.405 M=20 35.951</td>
</tr>
<tr>
<td>Static</td>
<td>( O(M) )</td>
<td>M=2 0.0155 M=5 0.060 M=10 0.060 M=15 0.060 M=20 0.0601</td>
</tr>
</tbody>
</table>

In the first example, we show the throughput performance of different algorithms versus SNR for \( M = 10 \) in Fig. 3.4. The objective of the dual problem (Upper-Bound) is also displayed in the same figure. We observe that the gap between the primal objective and the dual objective, i.e., the duality gap is close to zero for all SNR region, which validates the optimality of the proposed scheme. Moreover it can be seen that JointOpt yields the best performance over all SNR values. We notice a performance gain of 2.4 dB over the Static solution at rate equal to 1 bits/sec/Hz, and it increases to 2.85 dB at rate equal to 1.8 bits/sec/Hz. In comparison the rate
losses of the suboptimal algorithm is 0.6 dB and 1 dB, respectively. We observe that the SolWOP exhibits a slightly lower performance to the SubOpt over all SNR region but with a much higher complexity.

Next we examine the performance of the end-to-end rate versus the number of MUs. The corresponding curves at SNR = 10 dB are shown in Fig. 3.5. It can be seen that JntOpt always yields the best performance, and a significant gain over Static is observed when the number of the users increases. This is because Static does not exploit multi-user diversity and the optimization becomes more significant while increasing the number of users. The performance of SubOpt and SolWOP also increases with the number of users due to the similar reasons and both exhibits much better gain over Static. On the other hand, we observe a significant increase in running time of JntOpt and SolWOP in table 3.1, when the number of users increases. The running times of both SubOpt and Static is much less than that of both JntOpt and SolWOP, and do not increase much with the increasing of the
3.7 Summary

In this chapter, we developed power and sub-carrier allocation algorithms for multi-user bidirectional relay networks. The objective was to maximize the total number of users.

To get a more insight into the performance gain achieved from tone matching, in the next example we compare JntOpt and SubOpt with SolWOP under the case when \(|h_m,1| > |h_m,2| \ldots > |h_m,K|, |\tilde{h}_m,1| < |\tilde{h}_m,2| \ldots < |\tilde{h}_m,K|, |g_m,1| > |g_m,2| \ldots > |g_m,K|, |\tilde{g}_m,1| < |\tilde{g}_m,2| \ldots < |\tilde{g}_m,K|, \forall m\). The throughput curves versus SNR for \(M = 2\) and \(M = 10\) are shown in Fig. 3.6. It can be seen that JntOpt and SubOpt yield good performance. However, SolWOP exhibits much worse performance as compared to that in Fig. 3.4 because the good channel in MAP is always paired with the bad channel in BCP when no pairing strategy is adopted.

**Figure 3.5: Throughput versus the number of users at SNR= 10 dB.**

In this chapter, we developed power and sub-carrier allocation algorithms for multi-user bidirectional relay networks. The objective was to maximize the total...
throughput of the system subject to the power and the sub-carrier constraints. Due to exclusive sub-carrier allocation constraints of the OFDMA system and the one-to-one MAP to BCP tone matching policy, the formulated problem was a mixed integer programming problem. The Lagrangian theory was exploited to efficiently solve the problem. Further, a suboptimal algorithm was designed to reduce the computational complexity. The low complexity algorithm follows a stepwise approach such that only one parameter is optimized in each step. The computational complexities of different algorithms are analysed. Numerical results demonstrated that the proposed algorithms significantly outperform other candidates.

Figure 3.6: Throughput versus SNR under anti-symmetric channels for $M = 2$ and $M = 10$, respectively.
Chapter 4

OFDM Resource Allocation for Parallel Relay Communication

In this chapter, we explore resource optimization problem in dual hop multi-carrier parallel relay networks. Power allocation at the source and the beamforming at the relay nodes are optimized such that all the relays transmit simultaneously over the second hop. Further, to complete the studies, an orthogonal transmission is also considered where the relays transmit in different time slots.

4.1 Introduction

The cooperative communication is known to enhance the performance of wireless systems. In a multi-relay network, different relay nodes coordinate their transmission and transfer the data to destination node which combines all the signals. We consider a parallel relay transmission between the transmitter and the destination. The OFDM based parallel relays can transmit data in different ways, i.e.,

- The OFDMA transmission where each relay occupies a unique set of orthogonal sub-carrier and transmit simultaneously in the second hop. We have considered this transmission strategy in chapter 2 for multi-user multi-relay scenario.
4.1 Introduction

- Non-orthogonal transmission where all the relays transmit simultaneously over all the sub-carriers. In this work we consider this scheme of transmission.

- The orthogonal transmission such that each relay transmits in exclusive time slot over all OFDM sub-carriers. This resource allocation under this scheme is also discussed in this chapter.

The resource optimization in dual hop relay networks has been studied in [56]–[62]. The authors in [56] investigated the power allocation problem in parallel relay networks such that each relay transmits in a pre-assigned time slot under AF protocol. Moreover, a relay selection scheme was also proposed in this work. The power optimization in multi-relay system where the source node and each relay station transmit over orthogonal channels was considered in [57]. In flat fading transmission, the beamforming problem in AF multi-relay networks under an aggregate power constraint has been studied in [58]. The optimal beamforming coefficients under individual power constraints were derived in [59]. Distributed beamforming strategies were developed in [60]. [61] considered the distributive beamforming design in multi-relay network with multiple sources and multiple destinations. Further, [62] investigated the joint power loading and bandwidth allocation strategies for orthogonal and non-orthogonal multi-relay systems.

The relay networks with combination of OFDM transmission have been proposed to further enhance the system performance [20]–[21]. The relay power allocation problem to maximize the system throughput in OFDM based parallel relay networks has been considered in [63]. Further, the authors also investigated the relay power allocation in the scenario where each relay transmits in preassigned time slot in [64] and [65]. However, [63]–[65] did not consider the power allocation at the source node. In multi-hop networks the channels over different hops vary independently. With this fact, the sub-carrier paring in multi-relay systems has been considered in [16], [26], and [66]. However, all of these three works assumed an OFDMA based relay selection protocol where each sub-carrier can be assigned to maximum one relay station in each hop and each relay transmits over disjoint set
4.2 System Model

Consider a cooperative relay system where a source node (SN) communicates with the destination node (DN) through $N$ parallel RSs as shown in Fig.4.1. We adopt a dual hop transmission such that SN transmits in the first hop and the RSs transmit in the second hop. Further, we assume that each node is equipped with only one antenna. OFDM is applied and the available bandwidth is divided into $K$ sub-carriers. Moreover, we assume that SN and DN are located far from each other such that there is no direct communication link [17].

In the first time slot, SN transmits symbol $s_k$ over sub-carrier $k$ with power $p_k$. The signal received at the $n$-th RS is

$$y_{n,k}^{RS} = \sqrt{p_k}h_{n,k}s_k + z_{n,k},$$

(4.1)

where $h_{n,k}$ denotes corresponding channel coefficient and $z_{n,k}$ is the additive white Gaussian noise with variance $\sigma_{n,k}^2$. We apply sub-carrier pairing over two hops such of sub-carriers.

In this work, we consider an OFDM based single user multi-relay network with dual hop AF transmission. We first design a resource allocation scheme in a non-orthogonal transmission such that the sub-carrier pairing at the relay nodes, the power loading over different sub-carriers at the source node, and the beamforming at the relay nodes are optimized jointly to maximize the end-to-end system throughput. Later, we consider the TDMA transmission and develop a resource allocation strategy to maximize the throughput over two hops.

The rest of this chapter is organized as follows. In Section 4.2, we present the system model of multi-user two-way relay transmission and formulate the joint resource allocation problem. In Section 4.3, we develop the joint solution as well as the suboptimal method. The resource allocation algorithm under orthogonal transmission is studied in Section 4.4. Simulation results are presented in Section 4.5 and the chapter summary is reported in Section 4.6.
that the signal received over the $k$-th sub-carrier will be forwarded to DN over the $j$-th sub-carrier.

4.3 Cooperative Non-Orthogonal Transmission

In this section, we assume that in the first time slot all the relay nodes receive data from the SN over all the sub-carriers such that each relay occupy the same copy of source data. In the second time slot, all the relays simultaneously transmit the information to the destination node.

Denote the channel coefficient between the $n$-th RS and DN on the $j$-th sub-carrier as $g_{n,j}$. Denote $w_{n,j}$ as the complex beamforming weight applied at the $n$-th RS over the $j$-th sub-carrier which is to be designed later. The signal
4.3 Cooperative Non-Orthogonal Transmission

received at DN over sub-carrier pair \((k, j)\) is

\[
y_{DN}^{(k,j)}(k;j) = \sum_{n=1}^{N} g_{n,j} w_{n,j} h_{n,k} \sqrt{p_k} s_k + \sum_{n=1}^{N} g_{n,j} w_{n,j} z_{n,k} + z_d,
\]

where \(z_d\) represents the additive noise at DN with the variance \(\sigma_d^2\).

The SNR received over sub-carrier pair \((k, j)\) is given by [58]

\[
\text{SNR}_{(k,j)} = \frac{\left(\sum_{n=1}^{N} g_{n,j} w_{n,j} h_{n,k}\right)^2 p_k}{\sum_{n=1}^{N} |w_{n,j} g_{n,j}|^2 \sigma_r^2 + \sigma_d^2}.
\]  

To assist the mathematical formulation, we introduce a binary variable \(\pi_{(k,j)} \in \{0, 1\}\) for the sub-carrier pairing, such that \(\pi_{(k,j)} = 1\) if the \(k\)-th sub-carrier of the first hop is paired with the \(j\)-th sub-carrier of the second hop, and \(\pi_{(k,j)} = 0\) otherwise.

The overall system throughput can then be expressed as

\[
C = \frac{1}{2} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \log_2(1 + \text{SNR}_{(k,j)}).
\]  

Since each sub-carrier in the first hop can be coupled with one and only one sub-carrier in the second hop and vice versa, the sub-carrier pairing constraint can be expressed as

\[
\pi_{(k,j)} \in \{0, 1\}, \forall k, j; \sum_{k=1}^{K} \pi_{(k,j)} = 1, \forall j; \sum_{j=1}^{K} \pi_{(k,j)} = 1, \forall k.
\]  

Further, the power constraints are defined as

\[
\sum_{k=1}^{K} \sum_{j=1}^{K} \sum_{n=1}^{N} \pi_{(k,j)} |w_{n,j}|^2 (p_k |h_{n,k}|^2 + \sigma_r^2) \leq P_R,
\]

\[
\sum_{k=1}^{K} p_k \leq P_S.
\]

Our aim is to jointly optimize the power allocation at source node, the beamforming coefficients at each relay node, and the sub-carrier pairing at relay nodes such that the overall system throughput is maximized under the power constraints at the source and relay nodes.
The optimization can be stated as

\[
\max_{\pi, p, w} C, \quad \text{s.t.} \quad (4.5), (4.6), (4.4),
\]

with \( \pi = \{\pi_{(k,j)}\} \), \( p = \{p_k\} \), \( w = \{w_{n,j}\} \), and \( p_k \geq 0 \), \( w_{n,j} \geq 0 \), for all \( n = \{1, ..., N\} \), \( k = \{1, ..., K\} \), \( j = \{1, ..., K\} \).

### 4.3.1 Joint Optimization Algorithm

Finding the power variables \( p_k \) and the beamforming weights \( w_{n,j} \) together seems difficult. Fortunately, the structure of the problem permits to first compute the optimal beamforming coefficients for a given source power \( p_k \). Denote \( \rho_j \) as the total power allocated to sub-carrier \( j \) such that \( \sum_{j=1}^{K} \rho_j \leq P_R \). Then (4.7) can be re-stated as

\[
\max_{\pi, p, w} C
\]

subject to

\[
\sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \sum_{n=1}^{N} |w_{n,j}|^2 (p_k |h_{n,k}|^2 + \sigma_r^2) \leq \sum_{j=1}^{K} \rho_j, \\
\sum_{j=1}^{K} \rho_j \leq P_R, \quad (4.6), (4.4).
\]

Thus, for any sub-carrier pair \( (k, j) \) with known \( p_k \) the optimum beamforming problem can be stated as

\[
\max_{w} \log_2(1 + \text{SNR}_{(k,j)})
\]

subject to

\[
\sum_{n=1}^{N} |w_{n,j}|^2 (p_k |h_{n,k}|^2 + \sigma_r^2) \leq \rho_j.
\]

Since the logarithm is a monotonically increasing function, maximization of the objective in (4.9) is equivalent to the maximization of (4.2). Defining

\[
\psi = \sqrt{\frac{\gamma_{n,k} \gamma_{n,j}}{1 + \gamma_{n,j}}}, \quad \xi = \frac{1 + \gamma_{n,k} + \gamma_{n,j}}{1 + \gamma_{n,k}},
\]

with

\[
\gamma_{n,k} = |h_{n,k}|^2 p_k \sigma_r^2, \quad \text{and} \quad \gamma_{n,j} = |g_{n,j}|^2 \rho_j \sigma_d^2,
\]

the problem in (4.9) can be written as
\[
\max_w \frac{\sum_{n=1}^N |w_{n,j}|^2}{\sum_{n=1}^N |w_{n,j}|^2 \xi} \quad (4.12)
\]
s.t. \[
\sum_{n=1}^N |w_{n,j}|^2 (p_k |h_{n,k}|^2 + \sigma_r^2) \leq \rho_j. \quad (4.13)
\]

The problem becomes similar to the optimization in [58] and with some mathematical calculations, the the optimal value of \( w_{n,j} \) can be computed as
\[
\hat{w}_{n,j} = \frac{\sqrt{p_k \delta^*_{n,k} g_{n,j}^*}}{(\sigma_r^2 |g_{n,j}|^2 + \frac{\eta_{n,k}}{\rho_j}) \left( \sum_{n=1}^N \frac{|h_{n,k}|^2 |g_{n,j}|^2 \eta_{n,k}}{\sigma_d^2 (\sigma_r^2 |g_{n,j}|^2 + \frac{\eta_{n,k}}{\rho_j})^2} \right)}, \quad (4.14)
\]
where \( \eta_{n,k} \triangleq \sigma_d^2 (p_k |h_{n,k}|^2 + \sigma_r^2) \) and \((.)^*\) denotes the complex conjugate.

Substituting \( w_{n,j} = \hat{w}_{n,j} \) in (4.2) yields
\[
\text{SNR}_{k,j} = \sum_{n=1}^N \frac{p_k |h_{n,k}|^2 |g_{n,j}|^2}{p_k |h_{n,k}|^2 \sigma_d^2 + \rho_j |g_{n,j}|^2 \sigma_r^2 + \sigma_d^2 \sigma_r^2}. \quad (4.15)
\]
Thus, the problem (4.7) becomes
\[
\max_{\pi, p, \rho} \frac{1}{2} \sum_{k=1}^K \sum_{j=1}^K \pi_{(k,j)} \log_2 (1 + \text{SNR}_{k,j}) \quad (4.16)
\]
s.t. (4.6), (4.4), \( \sum_{j=1}^K \rho_j \leq P_R \),

where \( \rho = \{\rho_j\}, \forall j \).

The Lagrangian of (4.16) under power constraints can be written as
\[
L (p, \rho, \pi, \mu, \lambda) = \sum_{k=1}^K \sum_{j=1}^K \frac{\pi_{(k,j)}}{2} \log_2 \left( 1 + \text{SNR}_{(k,j)} \right) + \mu \left( P_S - \sum_{k=1}^K p_k \right) + \lambda \left( P_R - \sum_{j=1}^K \rho_j \right),
\]
where \( \mu \) and \( \lambda \) are the Lagrange multipliers associated with the corresponding power constraints. The dual function of (4.16) can be expressed as [13]
\[
D (\mu, \lambda) = \max_{\pi, p, \rho} L (p, \rho, \pi, \mu, \lambda), \quad \text{s.t. (4.4)}. \quad (4.17)
\]
4.3 Cooperative Non-Orthogonal Transmission

We adopt Lagrange dual decomposition approach to obtain the optimum dual function for the given dual variables $\mu$ and $\lambda$. The problem in (4.17) can be rewritten as

$$D(\mu, \lambda) = \max_{\pi, p, \rho} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \left( \frac{1}{2} \log_2 \left( 1 + \frac{\text{SNR}_{(k,j)}}{\rho_j} \right) - \mu p_k - \lambda \rho_j \right)$$

$$+ \mu P_s + \lambda P_R, \quad \text{s.t.} \quad (4.4). \quad (4.18)$$

Clearly, for the given sub-carrier pairing, i.e., for $\pi_{k,j} = 1$, at relatively high SNR (4.18) is decomposed into the following independent sub-problems

$$\max_{p_k, \rho_j} \frac{1}{2} \log_2 \left( 1 + \sum_{n=1}^{N} \frac{a_{n,k} p_k b_{n,j} \rho_j}{a_{n,k} p_k + b_{n,j} \rho_j} \right) - \mu p_k - \lambda \rho_j, \quad (4.19)$$

s.t. $p_k \geq 0$, $\rho_j \geq 0$, where $a_{n,k} \triangleq \frac{|h_{n,k}|^2}{\sigma_n^2}$ and $b_{n,j} \triangleq \frac{|g_{n,j}|^2}{\sigma_d^2}$.

Problem (4.19) is now in convex form and thus we refer to KKT conditions. Let $J$ be the Lagrangian associated with the sub-problem (4.19) with $\alpha_k$ and $\beta_j$ being the Lagrange multipliers for the constraints $p_k \geq 0$ and $\rho_j \geq 0$, respectively. Taking the derivative of $J$ w.r.t. $p_k$ and using the KKT conditions $\frac{\partial J}{\partial p_k} = 0$, $\alpha_k p_k = 0$, and $\alpha_k \geq 0$ we obtain

$$\mu = \frac{\left( \sum_{n=1}^{N} \frac{a_{n,k} \rho_j^2 b_{n,j}^2}{(p_k a_{n,k} + \rho_j b_{n,j})^2} \right)}{1 + \sum_{n=1}^{N} \frac{p_k a_{n,k} \rho_j b_{n,j}}{p_k a_{n,k} + \rho_j b_{n,j}}}. \quad (4.20)$$

Similarly, $\frac{\partial J}{\partial \rho_j} = 0$, $\beta_j \rho_j = 0$, and $\beta_j \geq 0$ yield

$$\lambda = \frac{\left( \sum_{n=1}^{N} \frac{b_{n,j}^2 a_{n,k}^2}{(p_k a_{n,k} + \rho_j b_{n,j})^2} \right)}{1 + \sum_{n=1}^{N} \frac{p_k a_{n,k} \rho_j b_{n,j}}{p_k a_{n,k} + \rho_j b_{n,j}}}. \quad (4.21)$$

Finding a closed form expression for $p_k$ and $\rho_j$ is difficult. However, solving (4.20) and (4.21) we obtain an intermediate expression

$$\rho_j = \frac{1}{\lambda} \left[ \frac{\sum_{n=1}^{N} \frac{a_{n,k}^2 \rho_j^2 b_{n,j}}{(p_k a_{n,k} + \rho_j b_{n,j})^2}}{\sum_{n=1}^{N} \frac{a_{n,k}^2 b_{n,j}^2}{(p_k a_{n,k} + \rho_j b_{n,j})^2}} \right] T(\rho_j). \quad (4.22)$$
4.3 Cooperative Non-Orthogonal Transmission

For a known $p_k$, the structure of $T(\rho_j)$ motivates us to apply the simple fixed point iterative algorithm [67].

Firstly, observe the following properties of $T(\rho_j)$. For $\rho_j > 0$,

- Positivity: $T(\rho_j) > 0$.
- Monotonicity: if $\rho_j > \rho_j'$, then $T(\rho_j) > T(\rho_j')$.
- Scalability: For all $\alpha > 1$, $\alpha T(\rho_{(k,j)}) > T(\alpha \rho_{(k,j)})$.

Thus, the fixed point iterations

$$
\rho_j^{(i+1)} = \sqrt{\frac{1}{\lambda} T(\rho_j^{(i)})},
$$

(4.23)

will converge to a unique value, denoted as $\bar{\rho}_j$ [67]. Now that we have obtained the optimum value $\bar{\rho}_j$ for some given value of $p_k$. Now for the obtained $\bar{\rho}_j$, we can easily find the optimum value of $p_k$, denoted as $\bar{p}_k$, through gradient descent method [13]. Finally, an alternate optimization over $\bar{p}_k$ and $\bar{\rho}_j$ is considered and at convergence the solutions are denoted by $\hat{p}_k$ and $\hat{\rho}_j$.

Substituting $\hat{p}_k$ and $\hat{\rho}_j$ into (4.18) yields

$$
D(\nu, \lambda) = \max_{\pi} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \Delta_{(k,j)} + \mu P_s + \lambda P_R
$$

s.t. (4.4),

(4.24)

where $\Delta_{(k,j)}$ represents the objective value in (4.19) at $\hat{p}_k$ and $\hat{\rho}_j$ where the analytical expression for $\Delta_{(k,j)}$ is missing due to unavailability of closed-form solutions for $\hat{p}_k$ and $\hat{\rho}_j$. We define a matrix with row indices being sub-carriers over the first hop and column indices being sub-carriers over the second hop. The values of each entry is the throughput obtained by pairing the corresponding sub-carriers. Problem (4.24) is equivalent to maximizing the sum throughput by choosing the best

$^{1}$The relay power $\bar{\rho}_j$ could also be obtained through gradient descent search, however fixed point algorithm in (4.23) significantly reduces the number of iterations require for convergence, as will be shown later in the simulations.
4.3 Cooperative Non-Orthogonal Transmission

A matching strategy that can only match one sub-carrier at the first hop to exactly one sub-carrier over the second hop. We find the optimal matching strategy from Hungarian algorithm [39]. Thus, the optimum sub-carrier pairing $\hat{\pi}$ is obtained.

Next we need to find the optimal dual variables from the following dual problem

$$\min_{\mu \geq 0, \lambda \geq 0} D(\mu, \lambda).$$

(4.25)

From the sub-gradient method [13], we could pick up any initial dual variables $\lambda^{(0)}$ and $\mu^{(0)}$. Then the dual variables at the $(t + 1)$-th iteration should be updated from

$$\mu(t + 1) = [\mu(t) - \delta(t)\Psi_\mu^+]^+, \lambda(t + 1) = [\lambda(t) - \delta(t)\Psi_\lambda^+]^+,$$

(4.26)

where $\Psi_\mu^+ \triangleq P_S - \sum_{k=1}^K \hat{p}_k(t)$, $\Psi_\lambda^+ \triangleq P_R - \sum_{k=1}^K \hat{p}_j(t)$, and $\delta(t)$ is an appropriate step size of the $t$-th iteration. Note that for each iteration, all the variables $\hat{\pi}(k,j)$, $\hat{p}_k$ and $\hat{p}_j$ should be re-computed under $\lambda(t)$ and $\mu(t)$. The iterations stop once certain criterion is fulfilled.

4.3.2 Suboptimal Algorithm

The resource allocation scheme presented in previous subsections gives a near optimal solution for a large number of sub-carriers. However, the computational complexity increases with the increasing of $K$. Thus, we refer to a suboptimal solution which trades the performance for lower complexity. The proposed suboptimal scheme solves optimization (4.7) in a step-wise fashion where each resource is optimized while fixing the others.

The steps of the suboptimal algorithm are outlined as

**Uniform Power Allocation**

Initially, we fix the power allocation by equally distributing the available powers to the $K$ sub-carriers, i.e., $p_k = \frac{P_S}{K}, \forall k$, $\rho_j = \frac{P_R}{K}, \forall j$ and the beamforming coefficients $w_{n,j}, \forall n, j$ are then found from (4.14).
4.3 Cooperative Non-Orthogonal Transmission

Subcarrier Pairing under Fixed Power Allocation

We define a $K \times K$ matrix $\Gamma$, where the $(k,j)$-th entry is

$$\Gamma_{(k,j)} = \frac{1}{2} \log 2 \left( 1 + \sum_{n=1}^{N} p_k |h_{n,k}|^2 \rho_j |g_{n,j}|^2 \right).$$

Our aim is to maximize the overall throughput such that a particular sub-carrier at hop-1 is paired with one and only one sub-carrier over hop-2. The different steps of algorithm are

1: Choose the optimal pairing $(k^*, j^*)$ such that

$$(k^*, j^*) = \max(\Gamma_{(k,j)}), \forall (k,j),$$

where $\Gamma_{(k,j)}$ denote the $(k,j)$-th entry in $\Gamma$,

2: Remove the $k^*$-th row and $j^*$-th column from $\Gamma$.

3: Repeat step-1 and step-2 until we get the complete optimum pairing set $\pi^*$.

Power Refinement Under Fixed Pairing

To obtain a more close performance to the joint resource allocation scheme, we re-compute the power allocation for the obtained sub-carrier pairing in step 2. Assume that the sub-carrier $k$ is paired with the sub-carrier $m(k)$, where $m$ is the mapping function. The problem in (4.18) becomes

$$D(\mu, \lambda) = \max_{p,\rho} \sum_{k=1}^{K} \left( \frac{1}{2} \log_2 \left( 1 + \text{SNR}_k \right) - \mu p_k - \lambda \rho_j \right) + \mu P_S + \lambda P_R,$$

(4.27)

where

$$\text{SNR}_k = \sum_{n=1}^{N} \frac{a_{n,k} p_k b_{n,m(k)} \rho_{m(k)}}{a_{n,k} p_k + b_{n,j} \rho_{m(k)} + 1}. \quad (4.28)$$

The problem (4.27) can be decomposed into $K$ sub-problems, each being similar to (4.19), and the solution for per sub-carrier problem follows the same steps in (4.19) to (4.23). Finally, the dual variables can be found from sub-gradient method.
4.4 Optimization under Orthogonal Transmission

4.3.3 Complexity Analysis

In subsection 4.3.1, we decompose the power allocation problem into $K^2$ sub-problems. The complexity of solving each sub-problem is $O(I(I' + I''))$, where $I'$ is the number of iterations required for the convergence of fixed point algorithm in (4.23), $I''$ is the number of iterations for finding $\bar{p}_k$ by gradient descent method, and $I$ denotes the number of iterations for alternate optimization over $\bar{p}_k$ and $\bar{p}_j$. Further the complexity of obtaining sub-carrier pairing through Hungarian algorithm is $O(K^3)$. If the objective of dual function is minimized in $M$ iterations, the total computational complexity of our proposed algorithm becomes $O(MK^2(I' + I'' + K))$.

On the other hand, in suboptimal algorithm the complexity of finding sub-carrier pairing is $O(K)$, and the power allocation in step 3 requires a complexity of $O(K\bar{I}(\bar{I}' + \bar{I}''))$, where $\bar{I}'$, $\bar{I}''$ and $\bar{I}$ are the number of iterations similar to $I'$, $I''$ and $I$, respectively. If the sub-gradient algorithm converges in $\bar{M}$ iterations, the total complexity involves in step 1 to step 3 in subsection 4.3.2 is $O(\bar{M}K(\bar{I}' + \bar{I}'' + 1))$.

Without loss of generality, we can consider $\bar{M}$, $\bar{I}$, $\bar{I}'$, and $\bar{I}''$ are close to $M$, $I$, $I'$, and $I''$, respectively due to the similarity between the algorithms. Then the overall complexity of the proposed suboptimal algorithm is much less than $O(MK^2(I' + I'' + K))$, the complexity of the joint resource allocation scheme.

4.4 Optimization under Orthogonal Transmission

In this section, an orthogonal transmission through time division is assumed where each node transmits in a preassigned time slot. The transmission from the source to the destination takes $N + 1$ time slots such that the source node transmits in the first time slot and then each RS transmits in subsequent time slots. The destination node combines the signals from all the relays through maximum ratio combining (MRC).

Based on the proposed protocol, the received SNR at the destination is given
by [56]

\[
\text{SNR}_{(k,j)} = \sum_{n=1}^{N} \frac{a_{n,k}p_k b_{n,j} q_{n,j}}{a_{n,k}p_k + b_{n,j} q_{n,j} + 1},
\]

(4.29)

with \( a_{n,k} \triangleq \frac{|h_{n,k}|^2}{\sigma_{a_{n,k}}^2} \) and \( b_{n,j} \triangleq \frac{|g_{n,j}|^2}{\sigma_{b_{n,j}}^2} \). The system throughput can be stated as

\[
C = \frac{1}{N + 1} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \log_2(1 + \text{SNR}_{(k,j)}).
\]

(4.30)

Further, the power constraints are defined as

\[
\sum_{k=1}^{K} p_k \leq P, \quad \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} q_{n,j} \leq Q_n.
\]

(4.31)

Finally, the optimization can be formulated as

\[
\max_{\pi, p, q} C, \quad \text{s.t.} \quad (4.4), (4.31).
\]

(4.32)

With (4.4), the problem (4.32) is a mixed integer programming and we solve it in the dual domain. The dual function can be defined as [13]

\[
D(\nu, \lambda) = \sum_{k=1}^{K} \sum_{j=1}^{K} \frac{\pi_{(k,j)}}{N + 1} \log_2(1 + \text{SNR}_{(k,j)}) + \lambda \left( P - \sum_{k=1}^{K} p_k \right)
\]

\[+ \sum_{n=1}^{N} \nu_n \left( Q_n - \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} q_{n,j} \right),
\]

(4.33)

where \( \lambda \) and \( \nu = \{\nu_1, \ldots, \nu_N\} \) are the Lagrange multipliers associated with the power constraints at the source and the relays, respectively. Similar to previous section, the optimal values of \( \nu \geq 0, \lambda \geq 0 \) which minimize the \( D(\nu, \lambda) \) could be obtained with the sub-gradient method [13].

For the given dual variables \( \nu \) and \( \lambda \), the problem in (4.33) can be rewritten as

\[
D(\nu, \lambda) = \max_{p, q, \pi} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \left( \frac{1}{N + 1} \log_2 \left( 1 + \text{SNR}_{(k,j)} \right) - \lambda p_k - \sum_{n=1}^{N} \nu_n q_{n,j} \right)
\]

\[+ \lambda P + \sum_{n=1}^{N} \nu_n Q_n, \quad \text{s.t.} \quad (4.4).
\]

(4.34)
4.4 Optimization under Orthogonal Transmission

Thus, for a sub-carrier pair \((k, j)\) with \(\pi_{k,j} = 1\), the problem is decomposed into following independent sub-problems

\[
\max_{p_k, q_j} \frac{1}{N+1} \log_2 \left( 1 + \text{SNR}_{(k,j)} \right) - \lambda p_k - \sum_{n=1}^{N} \nu_n q_{n,j} \tag{4.35}
\]

s.t. \(p_k \geq 0, q_{n,j} \geq 0, \forall n,\)

where \(q_j = \{q_{1,j}, q_{2,j}, \ldots, q_{N,j}\}\).

Let \(p^*_k\) and \(q^*_{n,j}\) be the optimal values of power variables which maximize the objective in (4.35). Substituting these values into dual function yields

\[
D(\nu, \lambda) = \max_{\pi} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \Delta_{(k,j)} + \lambda P + \sum_{n=1}^{N} \nu_n Q_n \tag{4.36}
\]

s.t. (4.4),

where \(\Delta_{(k,j)}\) is obtained by substituting \(p^*_k\) and \(q^*_{n,j}\) into (4.35). If \(\pi^*\) denotes the optimal sub-carrier pairing, the dual function in (4.33) is obtained.

It remains to find the per sub-carrier optimal power allocation and the sub-carrier pairing for the known power allocation.

(i) Optimal power allocation algorithm

The power allocation problem is

\[
\max_{p_k, q_j} \frac{1}{N+1} \log_2 \left( 1 + \sum_{n=1}^{N} \frac{a_{n,k}p_kb_{n,j}q_{n,j}}{a_{n,k}p_k + b_{n,j}q_{n,j}} \right) - \lambda p_k
\]

\[
- \sum_{n=1}^{N} \nu_n q_{n,j}, \text{ s.t. } p_k \geq 0, q_{n,j} \geq 0, \forall n. \tag{4.37}
\]

Since (4.37) is a convex problem, the KKT conditions can be used to obtain the optimal solution. We first obtain the power allocation at the relay nodes such that

\[
\tilde{q}_{n,j} = \frac{a_{n,k}p_k}{b_{n,j}} \left[ \sqrt{\frac{b_{n,j}}{(N+1)p_n} (X_n + a_{n,k}p_k) + \left( \frac{1}{2}a_{n,k}p_k \right)^2} - \left( X_n + \frac{1}{2}a_{n,k}p_k \right) \right] \quad \text{, } \tag{4.38}
\]
4.4 Optimization under Orthogonal Transmission

where

\[ X_n = \left( \sum_{i=1, i \neq n}^{N} \frac{a_{i,k}p_k b_{i,j}q_{i,j}}{a_{i,k}p_k + b_{i,j}q_{i,j}} + 1 \right). \]  

(4.39)

The detailed derivation steps are given in the Appendix A. Now for the obtained value of \( \bar{q}_j \), we can easily find \( \bar{p}_k \) (which is the optimal power at the source node for the obtained \( \bar{q}_j \)) through gradient descent method. Then the optimal values \( p^*_k \) and \( q^*_j \) are obtained from alternate optimization over \( \bar{p}_k \) and \( \bar{q}_j \).

(ii) Subcarrier Pairing Algorithm

Now it only remains to find the solution of sub-carrier pairing. Let \( S \) be a \( K \times K \) sub-carrier pairing matrix whose \( (k,j) \)-th entry is \( \Delta_{(k,j)} \). The optimum paring could be obtained from the Hungarian method, for a lower complexity we find a suboptimal pairing strategy similar to the previous section. In the first step, we choose the maximum valued entry in \( S \), whose index is denoted by \( (\hat{k}, \hat{j}) \). Then, remove the \( \hat{k} \)-th row and the \( \hat{j} \)-th column from \( S \). This process is repeated until we get the complete optimum pairing set \( \tilde{\pi} \).

Resource Optimization with Relay Selection

The resource optimization under orthogonal transmission enhances the system performance. However, the number of time slots required for a complete transmission increase with increasing the relay stations. Following the work in [56], we propose a relay selection scheme where a single best relay station is selected and a complete transmission between the source and destination takes only two time slots.

For the \( n \)-th relay station, we find the optimization

\[ C_n = \max_{p_k,q_{n,j}} \sum_{k=1}^{K} \sum_{j=1}^{K} \pi_{(k,j)} \frac{1}{2} \log_2 \left( 1 + \frac{a_{n,k}p_k b_{n,j}q_{n,j}}{a_{n,k}p_k + b_{n,j}q_{n,j}} \right) \]  

s.t. (4.4), \( \sum_{k=1}^{K} p_k \leq P, \sum_{j=1}^{K} q_{n,j} \leq Q_n. \)

\[ \[ (4.40) \] \]
Then, the overall system throughput is

$$C = \max_n C_n.$$  \hspace{1cm} (4.41)

The optimization problem (4.40) can be solved with similar steps as those in previous section. The main difference is in the per-sub-carrier optimal power allocation algorithm. For a given sub-carrier pairing \( \pi(k, j) = 1 \), the optimization becomes

$$\max_{p_k, q_{n,j}} \frac{1}{2} \log_2 \left( 1 + \frac{a_{n,k}p_k b_{n,j}q_{n,j}}{a_{n,k}p_k + b_{n,j}q_{n,j}} \right) - \lambda p_k$$

$$- \nu_n q_{n,j}, \; \text{s.t.} \; p_k \geq 0, \; q_{n,j} \geq 0.$$  \hspace{1cm} (4.42)

The structure of the above problem allows to find closed form expressions for \( p_k \) and \( q_{n,j} \). The KKT conditions yield

$$p_k^* = \frac{1}{\left( 1 + \sqrt{\frac{a_{n,k} \nu_n}{b_{n,j} \lambda}} \right) \frac{1 - \left( \sqrt{a_{n,k} \nu_n} + \sqrt{b_{n,j} \lambda} \right)^2}{a_{n,k} b_{n,j} \lambda}}^+, \; \hspace{1cm} (4.43)$$

$$q_{n,j}^* = \frac{1}{\left( 1 + \sqrt{\frac{b_{n,j} \lambda}{a_{n,k} \nu_n}} \right) \frac{1 - \left( \sqrt{a_{n,k} \nu_n} + \sqrt{b_{n,j} \lambda} \right)^2}{a_{n,k} b_{n,j} \nu_n}}^+. \; \hspace{1cm} (4.44)$$

The sub-carrier pairing algorithm and the solution of dual problem are similar to those in previous subsections and are omitted for simplicity.

### 4.5 Simulations

In this section, simulation results are provided to evaluate the performance of the proposed schemes. We assume 6-tap channels taken from i.i.d. Gaussian random variables for all links. The frequency domain channel is divided into 32 OFDM sub-carriers, i.e., \( K = 32 \). The noise variances at each RS and DN are set as \( \sigma_r^2 = \sigma_d^2 \).

**Non-orthogonal transmission**

First, we check the performance of our proposed algorithms for cooperative non-orthogonal transmission. We compare the joint power allocation and carrier
pairing (JPC) scheme, presented in subsections 4.3.1 with the proposed suboptimal power allocation and carrier pairing (SPC) algorithm in section 4.3.2. Moreover, we also show the performance of algorithm with suboptimal carrier pairing under equal power allocation (EPSC), i.e., only considering step-1 to step-2 in section 4.3.2. An algorithm with optimum power allocation where carrier pairing is not adopted (OPNC) is also considered. Finally, the heuristic where, without considering pairing, the power is equally distributed among sub-carriers (EPNC) is also included in the simulations.

Fig. 4.2 shows the performance comparison of five different algorithms discussed above. The system throughput versus SNR with $N = 10$ is shown, where the figure of the merit is taken as the per tone rate, i.e., sum rate divided by $K$. The objective of the dual problem (Upper), which serves as the upper bound, is also plotted in the same figure. We observe that the gap between the primal objective and the dual objective, i.e., the duality gap is close to zero for all SNR region, which validates the optimality of the proposed scheme. Moreover, it can be seen that JPC exhibits the
best performance over all schemes while SPC gives very near performance to JPC.

The convergence of the proposed algorithms is shown in Fig. 4.3. The maximum power per transmission are taken as $P_S = 10$, and $P_R = 15$. Fig. 4.3 (a) shows the convergence of the sub-gradient for JPC and OPNC algorithms. The abbreviations Sum-P, Sum-$\rho$ and ObjSub denote the total power allocated at source node (i.e., $\sum_{k=1}^{K} \hat{p}_k$), power allocated at relay nodes (i.e., $\sum_{j=1}^{K} \hat{p}_j$), and the value of objective function at $\hat{p}_k$ and $\hat{p}_j$, respectively. We see that the convergence is attained at 290 iterations. Further, at convergence both the power constraints $P_S$ and $P_R$ are satisfied, i.e., Sum-P and Sum-$\rho$ converges to the $P_S$ and $P_R$, respectively. Figure 4.3 (b) shows the convergence of per-sub-carrier pair power allocation algorithm, discussed in section 4.3.1. In the figure, FP and GD denote the fixed point algorithm and gradient descent algorithm, respectively, where FP-GD shows the convergence of alternate optimization over $\hat{p}_k$ and $\hat{p}_j$. It can be seen that the fixed point algorithm converges very fast only after few iterations, i.e., in 5 iterations, while the alternate optimization over $\bar{p}_k$ and $\bar{p}_j$ takes 12 iterations to converge. The gradient descent
4.5 Simulations

algorithm converges in 20 iterations, which are four times more than that of FP.

Orthogonal transmission

Under orthogonal transmission, we compare four different algorithms. 1) JPSC: The proposed joint power allocation and sub-carrier pairing solution. 2) Pow: An algorithm where optimum power allocation is obtained without considering sub-carrier pairing. 3) EPSC: A solution where the sub-carriers are paired according to the proposed algorithm under uniform power distribution such that the available power at each node is equally divided among all the sub-carriers. 4) EP: In this case, without considering sub-carrier pairing, each node equally distribute the available power to the $K$ sub-carriers. The throughput performance of four different methods versus SNR where the relays transmit in preassigned time slots is shown in Fig. 4.4. We have set $P = Q_n = 10, \forall n$. We observe that JPSC yields the best performance over all other algorithms. Further, the optimized power allocation solution (Pow) outperforms the uniform power allocation solutions. Optimizing only the sub-carrier pairing also provides performance gain in EPSC over the EP solution which validate the idea of performance achievement from sub-carrier matching over two hops. Finally, as expected, we observe that the system throughput decreases while increasing the number of relay nodes which is due to the lower TDMA factor $(\frac{1}{N+1})$ for higher $N$.

Next we examine the performance of the end-to-end rate versus the SNR under the relay selection scenario. The results for different algorithms are shown in Fig. 4.5, where the parameters are same as that of in Fig. 1. Similar to the previous example, JPSC yields the best performance among different methods and the proposed suboptimal methods (Pow, EPSC) also outperform the trivial solution (EP). Comparing the results with Fig. 4.4, all the schemes exhibit performance gain. Further, unlike the previous example, increasing the number of relay nodes improves the performance. This is consistent with the results in [56].
4.6 Summary

In this chapter, a dual hop communication was considered where the multiple relay nodes receive and transmit information over a common frequency band. First a non-orthogonal transmission was assumed where all the relays transmit over all the OFDM sub-carriers to maximize the system throughput. Different variables were optimized to develop a joint resource allocation algorithm, i.e., the power allocation over different sub-carriers at SN, the beamforming coefficients at RNs, and the sub-carrier pairing at the relays. The complexity of the joint resource allocation scheme increases with increasing the number of relay nodes. To reduce the computation burden, a suboptimal algorithm was then designed. Secondly, an orthogonal parallel transmission was studied where the relay nodes transmit in independent time slots. In this scenario, the power allocation at the source/relay nodes and the sub-carrier pairing was optimized to enhance the system throughput. However, this scheme provides much lower performance than the non-orthogonal scheme because of the $N + 1$ time slots required for a complete transmission. The
complexities of different schemes were compared. Numerical examples showed that the proposed algorithms significantly outperform the conventional solutions.
Chapter 5

Lifetime Maximization in Multi-hop Networks

In this chapter, we study the lifetime maximization problem in wireless multi-hop networks where a number of relaying nodes are interconnected in a series fashion to transfer the information from the source to the destination. Convex optimization techniques are utilized to maximize the total lifetime of the network.

5.1 Introduction

The development of advanced digital systems and wireless communication techniques have introduced a variety of wireless network technologies, such as wireless local area networks (LANs), home networks, and multi hop adhoc networks. Multi hop adhoc networks are groups of terminals that can communicate with each other without fixed infrastructure. Nodes may communicate directly or can use other nodes in the network as relays to facilitate the communication from the source to the destination. These networks can be rapidly deployed and are therefore well suited for the applications where a temporary network is required.

In multi-hop networks the nodes are usually resource constrained, e.g., they have limited energy and limited transmission range. A well known example is Wireless
5.1 Introduction

Sensor Networks (WSN), which typically consist of large number of nodes that have limited energy supply and are distributed in a certain area. These nodes collect and send certain information to a common sink in a hop by hop fashion as shown in Fig.5.1. Sensor nodes operate on small batteries and are generally deployed in an area where the replacement or recharging of batteries is not possible. Therefore, the life of a sensor network is usually defined by the time interval between which one or a certain amount of critical nodes run out of their battery energies.

A considerable research has been conducted to enhance the lifetime in WSNs. For example, optimum routing for lifetime maximization that is able to balance the load among different hops was designed in [68]-[69]. Maximum lifetime scheduling problem where the sensor nodes are deployed per unit area more denser than the required was considered in [70]. In multi-hop transmission, the closest nodes to the sink have more power consumption due to their relaying other nodes’ information [71]. Separate relay networks were proposed to release the relaying load over the sensor nodes in [72]-[73]. Moreover, [74] and the references therein considered the problem of relay node placement in WSNs which is known to increase the lifetime of the sensor nodes. However, it does not guarantee achieving the maximum lifetime of the relay nodes. The problem was further studied in [75], where mobile relays were used instead of fixed static relays.
5.1 Introduction

Figure 5.2: Linear multi-hop network

Recently, the authors in [76]-[79] considered the resource allocation problem in linear multi-hop networks. The so-called linear relay network consists of one dimensional chain of nodes including a source node, a destination node, and several intermediate relay nodes, as shown in Fig.5.2. It can be viewed as an important special case of a sensor network where only a single route is active. As a natural consequence of the multi-hop relaying and the OFDM transmission, the allocation of per-hop resources is crucial in optimizing the end-to-end network performance. The authors in [76] considered the optimum power allocation problem in linear multi-hop networks where each relaying node uses AF transmission mode, where DF mode of transmission was considered in [77]. The authors in [78] and [79] considered the per hop transmission time and the power allocation to maximize the system throughput and minimize the end-to-end outage, respectively. However, all the works in [76]-to-[79] considered the optimization problem under the system-wide power constraint. Such a power constraint may not be a good choice to see the performance in practical systems where all the nodes are located distributively.

In multi-hop relay networks, AF relays are generally not used because of the large noise enhancement. Hence, most literatures on resource allocation in multi-hop relay networks consider only the DF relays [78]. However, the use of DF relays requires energy consumption and time delay for decoding and encoding the message. We consider a new network model that consists of mixed AF-DF relay nodes. This new network structure halves the disadvantages of purely using AF or DF nodes. For the time being, we focus on the simplest alternating structure, that is, each DF relay node is preceded and followed by one AF relay node, as shown in Fig. 5.3.

In this chapter, we consider the lifetime maximization in AF-DF alternating
5.2 System Model

Consider a sensor relay network that is composed of one sensor node $S$, one sink node $D$ and $2N - 1$ intermediate relay nodes. The DF and the AF relay nodes will be indexed as $\text{DF}_n$ and $\text{AF}_n$, respectively, as shown in Fig. 5.3. Considering $S$ and $D$ as DF nodes, we can divide the $2N + 1$ nodes into $N$ sub-hops, with $n$th sub-hop consisting of nodes $\text{DF}_n$, $\text{AF}_n$, and $\text{DF}_{n+1}$. The $\text{DF}_n$ is a shared node of sub-hop $n$ and the sub-hop $n - 1$. We assume that each node is equipped with only one relay networks. We define the network lifetime as the minimum node lifetime, i.e., the time when the first node in the network runs out of energy. In order to cope with the general frequency selective fading channel, the OFDM modulation is adopted over each hop. We formulate the optimization problem as maximizing the network lifetime subject to the constraint of minimum end-to-end rate requirement, while each relay node is constrained to its limited battery energy as well as the maximum power supply. Under certain assumptions, we transform the original problem into the standard convex optimization problem and solve it from the dual decomposition technique. Simulation results are provided to corroborate the proposed strategy.

This chapter is organized as follows. In Section 5.2, we present the system model of multi-hop relay network. The lifetime maximization problem is formulated in Section 5.3 and the solution is proposed in Section 5.4. In Section 5.5, the numerical examples are presented and the chapter is summarized in the last section.

Figure 5.3: System model of AF-DF alternating relay network
5.2 System Model

antenna that cannot transmit and receive simultaneously. We also assume that each node can only receive the message sent from the immediately preceding node, i.e., a linear multi-hop topology is considered. Meanwhile, the OFDM with $K$ sub-carriers is adopted in each hop.

**Remark:** On average, $S$ is able to transmit a new symbol every two time slots. So, the time efficiency of such a multi-hop relay network is the same as the traditional dual hop relay network.

Denote the channel coefficients on the $k$th sub-carrier between $\text{DF}_n$ and $\text{AF}_n$ as $h_{n,k}$, and that between $\text{AF}_n$ and $\text{DF}_{n+1}$ as $g_{n,k}$, respectively. The signal received by $\text{AF}_n$ over sub-carrier $k$ can be written as

$$y_{n,k}^{\text{AF}} = p_{n,k}h_{n,k}x_{n,k} + n_{n,k}^{\text{AF}},$$

(5.1)

where $x_{n,k}$ and $p_{n,k}$ are the transmitted signal and the transmitted power on the $k$th sub-carrier from $\text{DF}_n$, and $n_{n,k}^{\text{AF}}$ is the additive noise received at $\text{AF}_n$ with variance $\sigma_{n,a}^2$. In the second time slot the received signal is re-transmitted by the AF relay after being scaled by factor $[80]$

$$\rho_{n,k} = \sqrt{\frac{q_{n,k}}{p_{n,k}|h_{n,k}|^2 + \sigma_{n,a}^2}},$$

where $q_{n,k}$ is the transmitted power of $\text{AF}_n$ over sub-carrier $k$. Then $\text{DF}_{n+1}$ receives

$$y_{n+1,k}^{\text{DF}} = \rho_{n,k}g_{n,k}p_{n,k}h_{n,k}x_{n,k} + \rho_{n,k}g_{n,k}n_{n,k}^{\text{AF}} + n_{n,k}^{\text{DF}},$$

where $n_{n,k}^{\text{DF}}$ denotes the noise received at DF relay node and has variance $\sigma_{n,d}^2$. Therefore, the throughput over sub-hop $n$ can be written as

$$C_n = \sum_{k=1}^{K} \frac{1}{2} \log_2 \left( 1 + \frac{|h_{n,k}|^2 \rho_{n,k}^2 |g_{n,k}|^2 p_{n,k}}{\rho_{n,k}^2 |h_{n,k}|^2 \sigma_{n,a}^2 + \sigma_{n,d}^2} \right),$$

(5.2)

with $a_{n,k} = \frac{|h_{n,k}|^2}{\sigma_{n,a}^2}$ and $b_{n,k} = \frac{|g_{n,k}|^2}{\sigma_{n,d}^2}$. Hence, the end-to-end system capacity can be defined as

$$\min_n C_n, \quad \forall \ n = 1, \ldots, N.$$  

(5.3)
5.3 Problem Statement

The constraint on the end-to-end performance can be written as

$$\min_n C_n \geq R,$$

where $R$ is the rate requirement.

5.3 Problem Statement

From previous discussion, the consumed powers over sub-hop $n$ from $\mathcal{DF}_n$ and $\mathcal{AF}_n$ are $p_n = \sum_{k=1}^K p_{n,k}$ and $q_n = \sum_{k=1}^K q_{n,k}$, respectively. Assume that the total battery energies of $\mathcal{DF}_n$ and $\mathcal{AF}_n$ are $E_{n}^{DF}$ and $E_{n}^{AF}$, respectively. Then the network lifetime is given by

$$T_{net} = \min \left( \frac{E_{1}^{DF}}{p_1}, \ldots, \frac{E_{N}^{DF}}{p_N}, \frac{E_{1}^{AF}}{q_1}, \ldots, \frac{E_{N}^{AF}}{q_N} \right).$$ \hspace{1cm} (5.4)

Our aim is to maximize the network lifetime by jointly optimizing the powers at each node such that a minimum end-to-end rate requirement is fulfilled. To make the discussion complete, we also assume that $\mathcal{DF}_n$ and $\mathcal{AF}_n$ have maximum power constraints as $P_n$ and $Q_n$, respectively, due to the linearity of the power amplifiers.

The optimization problem is finally formulated as

$$\max_{p_{n,k}, q_{n,k}} \quad T_{net}$$

s.t. $\min_n \left\{ \sum_{k=1}^K r_{n,k} \right\} \geq R$, \hspace{1cm} $\sum_{k=1}^K p_{n,k} \leq P_n$, \hspace{1cm} $\forall n$

$\sum_{k=1}^K q_{n,k} \leq Q_n$, \hspace{1cm} $\forall n$

$p_{n,k} \geq 0$, \hspace{1cm} $q_{n,k} \geq 0$, \hspace{1cm} $\forall n, k.$
5.4 Lifetime Maximization Scheme

Introducing an auxiliary variable $t$, we first reformulate the problem as

\[
\begin{align*}
\max_{p_{n,k}, q_{n,k}, t} & \quad t \\
\text{s.t.} & \quad t \leq \frac{E_n^{DF}}{\sum_{k=1}^{K} p_{n,k}}, \quad t \leq \frac{E_n^{AF}}{\sum_{k=1}^{K} q_{n,k}}, \quad \forall n \\
& \quad \sum_{k=1}^{K} r_{n,k} \geq R, \quad \sum_{k=1}^{K} p_{n,k} \leq P_n, \quad \forall n \\
& \quad \sum_{k=1}^{K} q_{n,k} \leq Q_n, \quad \forall n \\
& \quad p_{n,k} \geq 0, \quad q_{n,k} \geq 0, \quad \forall n, k.
\end{align*}
\]

Changing the variable $t = \frac{1}{z}$, we can reformulate the above problem into an equivalent standard convex optimization problem:

\[
\begin{align*}
\min_{p_{n,k}, q_{n,k}, z} & \quad z \\
\text{s.t.} & \quad \sum_{k=1}^{K} p_{n,k} - zE_n^{DF} \leq 0, \quad \forall n \\
& \quad \sum_{k=1}^{K} q_{n,k} - zE_n^{AF} \leq 0, \quad \forall n \\
& \quad R - \sum_{k=1}^{K} r_{n,k} \leq 0, \quad \sum_{k=1}^{K} p_{n,k} - P_n \leq 0, \quad \forall n \\
& \quad \sum_{k=1}^{K} q_{n,k} - Q_n \leq 0, \quad \forall n \\
& \quad z \geq 0, \quad p_{n,k} \geq 0, \quad q_{n,k} \geq 0 \quad \forall n, k,
\end{align*}
\]

where the first two constraints are linear in $z$.

In problem (5.7), we can always choose a large enough value of $z$ such that the energy constraints are satisfied with strict inequality. In addition, we assume that there always exists a feasible solution such that the rate and the maximum power constraints are satisfied with strict inequality (However, at optimality the rate constraint $R - \sum_{k=1}^{K} r_{n,k} \leq 0, \quad \forall n$ holds with equality). Thus the Slater’s
condition is satisfied [13]. Therefore, the strong duality holds and we can obtain the optimal primal solution by solving the corresponding dual problem [13] under the stated assumptions.

For optimization problem (5.7), the dual function can be defined as

\[
D(\lambda, \eta, \nu, \bar{\lambda}, \bar{\eta}) = \min_{p, n, k \geq 0, q, n, k \geq 0, z \geq 0} L(p, n, k, q, n, k, z, \lambda_n, \eta_n, \nu_n, \bar{\lambda}_n, \bar{\eta}_n)
\]

(5.8)

where

\[
L(p, q, z, \lambda, \eta, \nu, \bar{\lambda}, \bar{\eta}) = z + \sum_{n=1}^{N} \lambda_n \left( \sum_{k=1}^{K} p_{n,k} - zE_{n}^{DF} \right) + \sum_{n=1}^{N} \eta_n \left( \sum_{k=1}^{K} q_{n,k} - zE_{n}^{AF} \right)
+ \sum_{n=1}^{N} \nu_n \left( R - \sum_{k=1}^{K} r_{n,k} \right) + \sum_{n=1}^{N} \bar{\lambda}_n \left( \sum_{k=1}^{K} p_{n,k} - P_n \right)
+ \sum_{n=1}^{N} \bar{\eta}_n \left( \sum_{k=1}^{K} q_{n,k} - Q_n \right)
\]

(5.9)

The dual problem is

\[
\max_{\lambda, \eta, \nu, \bar{\lambda}, \bar{\eta}} D(\lambda, \eta, \nu, \bar{\lambda}, \bar{\eta})
\]

(5.10)

s.t. \( \lambda_n \geq 0, \ \eta_n \geq 0, \ \nu_n \geq 0, \ \bar{\lambda}_n \geq 0, \ \bar{\eta}_n \geq 0 \ \forall \ n. \)

To facilitate the immediate recovery of auxiliary variables \( z \), we change the primal objective function in (5.7) to \( z^2 \); since for \( z \geq 0 \) minimizing \( z \) is equivalent to minimizing \( z^2 \). Then the Lagrangian \( L \) in (5.9) can be re-written as

\[
L = \left( z^2 - \sum_{n=1}^{N} \left( \lambda_n E_{n}^{DF} + \eta_n E_{n}^{AF} \right) \right) + \sum_{n=1}^{N} \sum_{k=1}^{K} \left( (\lambda_n + \bar{\lambda}_n) p_{n,k} + (\eta_n + \bar{\eta}_n) q_{n,k} - \nu_n r_{n,k} \right)
+ \sum_{n=1}^{N} \left( \nu_n R - \bar{\lambda}_n P_n - \bar{\eta}_n Q_n \right),
\]

(5.11)

and the dual function becomes

\[
D(\lambda, \eta, \nu, \bar{\lambda}, \bar{\eta}) = \min_{z \geq 0} \left( z^2 - \sum_{n=1}^{N} \left( \lambda_n E_{n}^{DF} + \eta_n E_{n}^{AF} \right) \right) + \sum_{n=1}^{N} \left( \nu_n R - \bar{\lambda}_n P_n - \bar{\eta}_n Q_n \right)
+ \sum_{n=1}^{N} \sum_{k=1}^{K} \min_{p_{n,k}, q, n, k \geq 0} \left( \Lambda_n p_{n,k} + \Gamma_n q_{n,k} - \nu_n r_{n,k} \right)
\]

(5.12)
5.4 Lifetime Maximization Scheme

where $\Lambda_n \triangleq (\lambda_n + \bar{\lambda}_n)$ and $\Gamma_n \triangleq (\eta_n + \bar{\eta}_n)$.

Hence, for given values of dual variables $\lambda_n, \eta_n, \bar{\lambda}_n, \text{ and } \bar{\eta}_n, \forall n$, the original problem is decomposed into following $NK + 1$ simple sub-problems

$$\min_{z \geq 0} \left( z^2 - z \sum_{n=1}^{N} (\lambda_n E_n^{DF} + \eta_n E_n^{AF}) \right)$$

and

$$\min_{p_{n,k} \geq 0, q_{n,k} \geq 0} (\Lambda_n p_{n,k} + \Gamma_n q_{n,k} - \nu_n r_{n,k}), \quad \forall n, k.$$

These sub-problems can be solved by standard convex optimization techniques.

For given values of dual variables $\lambda_n$ and $\eta_n$, we can solve the sub-problem (5.13) and obtain

$$z^* = \frac{1}{2} \left( \sum_{n=1}^{N} (\lambda_n E_n^{DF} + \eta_n E_n^{AF}) \right).$$

The optimization problem (5.14) is a standard convex optimization problem in variables $p_{n,k}$ and $q_{n,k}$. Applying KKT conditions we can find the optimal values of $p_{n,k}$ and $q_{n,k}$ as

$$p_{n,k}^* = \frac{1}{\sqrt{a_{n,k} b_{n,k}} + \sqrt{\Lambda_n \Gamma_n}} \left[ \frac{\nu_n}{\sqrt{\Lambda_n \Gamma_n}} - \frac{\left( \sqrt{\frac{\Gamma_n}{\lambda_n}} a_{n,k} + \sqrt{\frac{\Lambda_n}{\Gamma_n}} b_{n,k} \right)^2}{a_{n,k} b_{n,k}} \right]^+$$

and

$$q_{n,k}^* = \frac{1}{\sqrt{a_{n,k} b_{n,k}} + \sqrt{\Lambda_n \Gamma_n}} \left[ \frac{\nu_n}{\sqrt{\Lambda_n \Gamma_n}} - \frac{\left( \sqrt{\frac{\Gamma_n}{\lambda_n}} a_{n,k} + \sqrt{\frac{\Lambda_n}{\Gamma_n}} b_{n,k} \right)^2}{a_{n,k} b_{n,k}} \right]^+$$

A detailed derivation of the solution can be found in Appendix B. Substituting (5.15)-(5.17) into (5.12) yields the dual function.

The dual function in (5.12) is convex and the sub-gradient method [40] can be used to solve the dual problem with guaranteed convergence. Using sub-gradient
method, the dual variables at \((i+1)\)th iteration are updated as

\[
\begin{align*}
\lambda_{n}^{i+1} &= \left( \lambda_{n}^{i} + \delta^{i} \left( \sum_{k=1}^{K} p_{n,k}^{*} - z^{*}E_{DF}^{n} \right) \right)^{+}, \forall n \\
\eta_{n}^{i+1} &= \left( \eta_{n}^{i} + \delta^{i} \left( \sum_{k=1}^{K} q_{n,k}^{*} - z^{*}E_{AF}^{n} \right) \right)^{+}, \forall n \\
\nu_{n}^{i+1} &= \left( \nu_{n}^{i} + \delta^{i} \left( R - \sum_{k=1}^{K} r_{n,k}^{*} \right) \right)^{+}, \forall n \\
\bar{\lambda}_{n}^{i+1} &= \left( \bar{\lambda}_{n}^{i} + \delta^{i} \left( \sum_{k=1}^{K} p_{n,k}^{*} - P_{n} \right) \right)^{+}, \forall n \\
\bar{\eta}_{n}^{i+1} &= \left( \bar{\eta}_{n}^{i} + \delta^{i} \left( \sum_{k=1}^{K} q_{n,k}^{*} - Q_{n} \right) \right)^{+}, \forall n
\end{align*}
\]

where \(\delta\) is appropriate step size and \(z^{*}, p_{n,k}^{*}, q_{n,k}^{*}, r_{n,k}^{*}\) are the optimum values of primal variables, computed at \(i\)th iteration from (5.15), (5.16), and (5.17).

The algorithm is summarized as follows:

1. Initialize dual variables \(\lambda_{n}, \eta_{n}, \nu_{n}, \bar{\lambda}_{n}, \bar{\eta}_{n}\).

2. Find the optimum values of primal variables using (5.15), (5.16), and (5.17).

3. Update dual variables using equations (5.18)-(5.22).

4. Repeat step 2 to 3 until convergence.

In the proposed algorithm we assume that complete channel state information (CSI) of sub-hop \(n\) is known to both \(\text{DF}_{n}\) and \(\text{AF}_{n}\). Therefore, \(p_{n,k}\) and \(q_{n,k}\) can be computed distributively at each \(\text{DF}_{n}\) and \(\text{AF}_{n}\). Similarly, each \(\text{DF}_{n}\) and \(\text{AF}_{n}\) can compute Lagrange multipliers \(\lambda_{n}, \eta_{n}, \bar{\lambda}_{n}, \bar{\eta}_{n}\) and \(\nu_{n}\) independently with the information from the previous iteration. However the computation of \(z\) requires values of Lagrange multipliers and values of initial energy at each node over all the hops. This can be done in a centralized fashion.
5.5 Simulation Results

In this section, we present simulation results to evaluate the performance of our proposed algorithm. We consider a network with 10 relay nodes such that there are 5 virtual sub-hops. We assume that all the transmitting nodes, i.e., source and relay nodes, have same initial energy and each node can provide equal amount of maximum power. We consider an OFDM system with \( K = 32 \) sub-carriers. Further we assume \( \sigma_{n,a}^2 = \sigma_{n,d}^2, \ \forall \ n \). The SNR is defined as the ratio between the transmitted signal power and the additive noise power. We compare the performance of proposed algorithm (OPT-SOL) with the equal power allocation solution (EP-SOL), where the available power on each node is evenly distributed among the \( K \) sub-carriers.

Fig. 5.4 shows the convergence of the proposed algorithm at SNR=30 dB. The initial energy of each node is taken as \( E_{n}^{AF} = E_{n}^{DF} = 50 \) and the maximum power per transmission is taken as \( P_n = Q_n = 2 \). The minimum end-to-end rate requirement (R-Req) over complete OFDM block is 64 bits/s/Hz. Obviously the system with EP-SOL has lifetime equal to 25 seconds. Fig. 5.4 (a) shows that lifetime \( t \) converges
5.5 Simulation Results

after 275 iterations and is equal to 138.6 seconds. Fig. 5.4 (b) shows the convergence of rates at first and the fifth hop. We see that, with convergence of variable $t$, the rate at each hop converges to the R-Req, which satisfies our statement in Section 5.4. For the clarity in figure, we showed the rate convergence only at two hops. The rates at other hops also converge in a similar way.

In Fig. 5.5, we look into the relationship of lifetime with rate constraint and initial energy. We observe that, there is always a trade off between network lifetime and R-Req. Increasing the minimum rate requirement decreases the lifetime. We observe that the slope of lines increases with the increasing of initial energy. Therefore the rate constraint is more crucial to the lifetime at high initial energy. Similarly we can see that at lower R-Req, increasing the initial energy is more beneficial to the network lifetime.

Fig. 5.6 (a) shows the network lifetimes at different SNR values. It can be seen that at low SNR the OPT-SOL does not perform well. The lifetime is much less than the EP-SOL. However before concluding, we look into Fig. 5.6 (b) which shows the minimum hop rate (R-min), i.e., the end-to-end network rate, provided

![Figure 5.5: Lifetime vs R-Req](image-url)
5.6 Summary

In this chapter, we defined and solved lifetime maximization problem in sensor relay networks. To suppress the disadvantages of using pure AF or DF relay modes, we considered a mixed AF-DF multi-hop network. We formulated the optimization problem considering both the initial energy and the power constraint at each node. With change of variables, the problem was reformulated to the standard convex optimization problem. We solved the problem using dual decomposition approach and presented the algorithm based on sub-gradient method. Convergence of the

Figure 5.6: Lifetime vs SNR

by the two schemes. We see that at low SNR both the schemes failed to provide the required rate. This means that there exists no feasible solution under the given set of constraints. Therefore, we consider only the high SNR region. We see that from SNR= 23 dB onwards, the R-min converges to the R-Req. At SNR= 30 dB, lifetime is 138.6 seconds which is quite high as compared to the EP-SOL.

5.6 Summary

In this chapter, we defined and solved lifetime maximization problem in sensor relay networks. To suppress the disadvantages of using pure AF or DF relay modes, we considered a mixed AF-DF multi-hop network. We formulated the optimization problem considering both the initial energy and the power constraint at each node. With change of variables, the problem was reformulated to the standard convex optimization problem. We solved the problem using dual decomposition approach and presented the algorithm based on sub-gradient method. Convergence of the
algorithm was shown by simulations. Through numerical results, we observed that at lower rate requirement, increasing the initial energy is more advantageous for network lifetime maximization.
Chapter 6

Resource Allocation in Cognitive Relay Networks

This chapter is available in the printed version and it has been removed from electronic version for copyright reasons.
Chapter 7

Conclusions

In this thesis, we have designed various resource allocation algorithms for OFDM based relay networks. Following are the major contributions of the thesis.

First, we have developed resource allocation algorithms to maximize the throughput of an OFDMA based uplink multi-user relay network. The algorithms optimized the sub-carrier allocation to the users, the power allocation at the sub-carriers, and the sub-carrier pairing over the two hops. Further, we extended the scheme to OFDMA based multi-relay network. The simulations results have shown the significant improvement with the developed schemes as compared to the existing schemes.

As a second step, we studied the resource allocation in TWRNs. A multi-user relay system with individual power constraint at each user and the relay node was proposed. The power optimization, sub-carrier assignment, and the tone matching were considered to enhance the throughput of a system that operates under OFDM based bidirectional communication. Efficient resource allocation schemes have been developed which demonstrated significant performance enhancement as compared to the trivial algorithms.

Third, we explored the resource allocation in dual hop multi-relay system. Under non-orthogonal transmission, optimal power allocation at the source node, the beamforming at the relay nodes, and the sub-carrier permutation over two hops
7. Conclusions

were considered for the maximization of end-to-end data rate. We developed an iterative joint resource optimization algorithm and validated the convergence and the performance though numerical examples. To reduce computational burden, a low complexity suboptimal scheme was also presented which demonstrated its comparable performance. Then, we looked into the problem under orthogonal transmission where each relay node transmits in an independent time slot. Further, a relay selection scheme was also developed which performs better than the orthogonal scheme. Numerical results corroborated the proposed schemes.

Next we presented the lifetime maximization problem in multi-hop energy limited sensor relay networks. The power allocation over different sub-carriers was optimized to maximize the end-to-end lifetime of the network. We formulated problem considering both the initial energy and the power constraint at each node in the network. Through numerical results, we observed that at lower rate requirement, increasing the initial energy is more advantageous for lifetime maximization.

Finally, we considered the resource allocation in OFDM based cognitive radio relay networks. We maximized the secondary user’s throughput while interference introduced to the primary user remain below a give limit. We explored the impact of sub-carrier pairing and the power allocation at the secondary user’s performance subject the power and interference constraints. Numerical results showed that the power allocation and the sub-carrier pairing algorithms developed for conventional OFDM relay networks perform worst when applied to the CR relay networks and the proposed algorithms outperform the trivial schemes.
Bibliography


101


Bibliography


List of Publications


Appendix A

Derivations of Closed-Form Expression for Power Allocation at the Relays

The Lagrangian $\bar{J}$ associated with (4.37) is

$$\bar{J} = \frac{1}{N+1} \log_2 \left( 1 + \sum_{n=1}^{N} \frac{a_{n,k}p_k b_{n,j} q_{n,j}}{a_{n,k}p_k + b_{n,j} q_{n,j}} \right) - \lambda p_k - \sum_{n=1}^{N} \nu_n q_{n,j} + \alpha_k p_k + \sum_{n=1}^{N} \beta_n q_{n,j},$$

(A.1)

where $\alpha_k$ and $\beta_n$ are the Lagrange multipliers associated with the power constraints in (4.37). Taking the derivative of $\bar{J}$ w.r.t. $q_{n,j}$ and setting $\frac{\partial \bar{J}}{\partial q_{n,j}} = 0$, we get

$$\beta_n = \nu_n - \frac{|a_{n,k}|^2 p_k^2 b_{n,j}}{(a_{n,k}p_k + b_{n,j} q_{n,j})^2} \left( \frac{a_{n,k}p_k b_{n,j} q_{n,j}}{(a_{n,k}p_k + b_{n,j} q_{n,j})^2} \right) = 0.$$  

(A.2)

From slackness condition, we obtain

$$q_{n,j} \left( \nu_n - \frac{|a_{n,k}|^2 p_k^2 b_{n,j}}{(a_{n,k}p_k + b_{n,j} q_{n,j})^2} \left( \frac{a_{n,k}p_k b_{n,j} q_{n,j}}{(a_{n,k}p_k + b_{n,j} q_{n,j})^2} \right) \right) = 0.$$  

(A.3)

Now, the KKT condition $\beta_n \geq 0$ along with (A.3) implies that for $q_{n,j} > 0$

$$\nu_n = \frac{|a_{n,k}|^2 p_k^2 b_{n,j}}{(a_{n,k}p_k + b_{n,j} q_{n,j})^2} \left( \frac{a_{n,k}p_k b_{n,j} q_{n,j}}{(a_{n,k}p_k + b_{n,j} q_{n,j})^2} \right) \left( \frac{a_{n,k}p_k b_{n,j} q_{n,j}}{(a_{n,k}p_k + b_{n,j} q_{n,j})^2} \right).$$  

(A.4)

and $q_{n,j} = 0$ otherwise. Solving (A.4) for $q_{n,j}$, we obtain expression in (4.38).
Appendix B

Derivations of the Optimal Power Allocations in Equations (5.16) and (5.17)

Taking the derivative of the Lagrangian associated with problem (5.14) and setting it to zero, we have

\[ \alpha_{n,k} = \Lambda_n - \frac{\nu_n a_{n,k} q_{n,k}^2 b_{n,k}^2}{(p_{n,k} a_{n,k} + q_{n,k} b_{n,k})} \times \frac{1}{(p_{n,k} a_{n,k} + q_{n,k} b_{n,k} + p_{n,k} a_{n,k} q_{n,k} b_{n,k})}. \]  

(B.1)

In the above equation, expanding the denominator term and applying KKT conditions, we obtain

\[ \Lambda_n \geq \frac{\nu_n a_{n,k} q_{n,k}^2 b_{n,k}^2}{(q_{n,k} b_{n,k})^2 + p_{n,k} (p_{n,k} a_{n,k}^2 + a_{n,k} q_{n,k} b_{n,k} (p_{n,k} a_{n,k} + q_{n,k} b_{n,k} + 2))}. \]  

(B.2)

and

\[ p_{n,k} \left( \Lambda_n - \frac{\nu_n a_{n,k} q_{n,k}^2 b_{n,k}^2}{(p_{n,k} a_{n,k} + q_{n,k} b_{n,k}) (p_{n,k} a_{n,k} + q_{n,k} b_{n,k} + p_{n,k} a_{n,k} q_{n,k} b_{n,k})} \right) = 0 \]  

(B.3)

(a) if \( \Lambda_n < \frac{\nu_n a_{n,k} q_{n,k}^2 b_{n,k}^2}{q_{n,k} b_{n,k}} \), then the condition (B.2) can only be fulfilled if \( p_{n,k} > 0 \) and in this case (B.3) can only be satisfied if

\[ \Lambda_n = \frac{\nu_n a_{n,k} (q_{n,k})^2 b_{n,k}^2}{(p_{n,k} a_{n,k} + q_{n,k} b_{n,k})} \times \frac{1}{(p_{n,k} a_{n,k} + q_{n,k} b_{n,k} + p_{n,k} a_{n,k} q_{n,k} b_{n,k})}. \]  

(B.4)
B. Derivations of the Optimal Power Allocations in Equations (5.16) and (5.17)

(b) if \( \nu > \frac{a_{n,k}(q_{n,k})^2 b_{n,k}^2}{(q_{n,k})^2 (b_{n,k})^2} \), with \( p_{n,k} > 0 \) it is impossible to fulfil (B.3) because in that case (B.2) holds with > sign and (B.3) couldn’t be satisfied, therefore in this case \( p_{n,k} = 0 \). Similarly for \( q_{n,k} > 0 \), we get

\[
\Gamma_n = \frac{\nu_n b_{n,k} p_{n,k} a_{n,k}^2}{(p_{n,k} a_{n,k} + q_{n,k} b_{n,k})^2} \times \frac{1}{(p_{n,k} a_{n,k} + q_{n,k} b_{n,k} + p_{n,k} a_{n,k} q_{n,k} b_{n,k})}. \tag{B.5}
\]

Otherwise, \( q_{n,k} = 0 \). Solving equations (B.4) and (B.5) for \( p_{n,k} \) and \( q_{n,k} \) respectively, we get the results in (5.16) and (5.17).